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1955

The Structure of the Developed Photographic Image
C. E. Kenneth Mees

On the Complex Representation of the General Extensional and Flexural Problems of Thin Plates and Their Analogies
Yi-Yuan Yu

The Present Status of Atmospheric Electricity
W. F. G. Swann

On the Ampère Force
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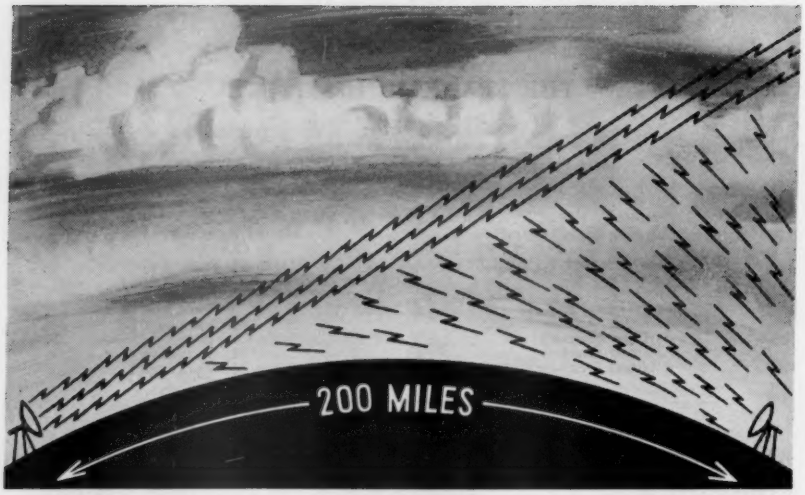
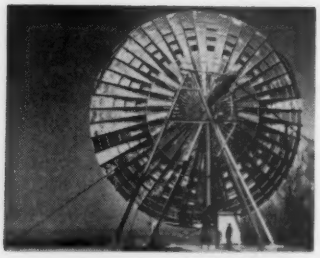


Highly schematic drawing illustrates the possible distribution of energy in ultra-high-frequency "over-the-horizon" transmission. The effect is similar to that of a powerful searchlight whose beam points into the sky. Light can be seen miles away from behind a hill even when the searchlight lens is invisible.

Something new on the telephone horizon



This experimental 60-foot antenna (rear view) photographed at the Bell Laboratories in Holmdel, New Jersey, is designed for study of "over-the-horizon" phenomena.

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Vol. 260 OCTOBER, 1955 No. 4

THE STRUCTURE OF THE DEVELOPED PHOTOGRAPHIC IMAGE

BY

C. E. KENNETH MEES[1]

When I was a boy studying chemistry, I was introduced to the theory of the molecule, and I thought I'd like to look at some molecules to see what they looked like. My desire has been realized because, by means of the X-ray, it is possible to do the equivalent of looking at molecules—at any rate, when they are arranged so that they form crystals—to see how the atoms that build up each molecule are arranged. In my youth, however, nothing was known about X-rays, and I thought I could see the molecules through a microscope.

When I discussed the subject with my schoolmaster, he explained that the microscope did not magnify enough to show molecules; that

FIG. 1. Negative at normal size.

even the best microscopes magnified only some 2,000 diameters, and that was not nearly enough to show molecules. I suggested that we should take a photograph through the microscope and then put this photograph under the microscope, so magnifying twice two thousand diameters, thereby getting four million diameters, and, if necessary, do it again and get eight thousand million diameters. My schoolmaster did not think it would be successful, but he was not very clear as to the reason. Let us see what happens if we try it.

If we magnify the photograph shown in Fig. 1 ten diameters, we see that the image is no longer quite smooth; it is beginning to look a little irregular (Fig. 2). We magnify it now 100 diameters instead of 10, and

[1] Vice President in charge of Research, Eastman Kodak Company, Rochester, N. Y.

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FIG. 2. Negative enlarged 10 times.

FIG. 3. Portion of negative enlarged 100 times.

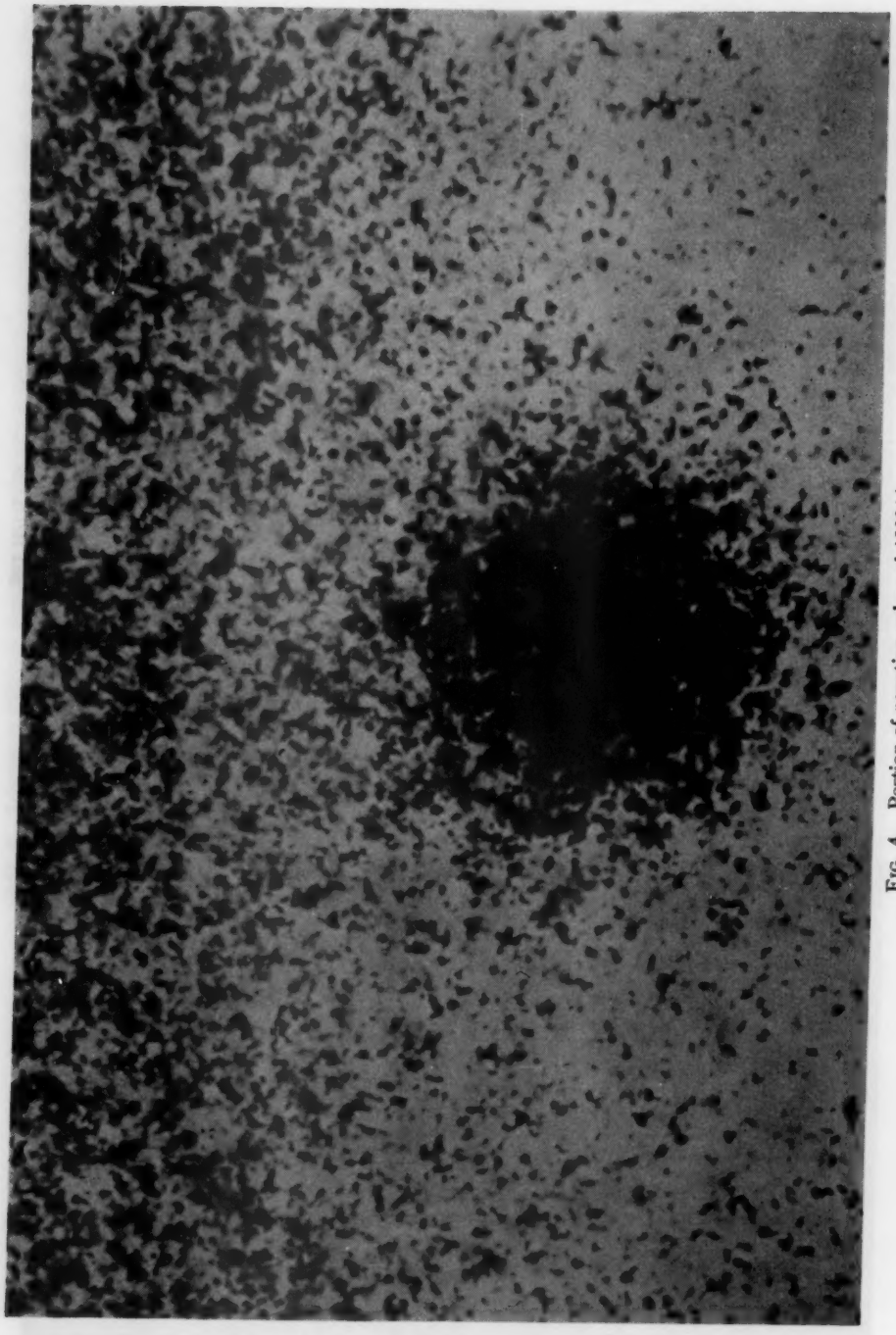

FIG. 4. Portion of negative enlarged 1000 times.

the image is not at all regular (Fig. 3). It consists obviously of speckly grains, and only with difficulty can we see that it represents a portion of the eye of the subject. If we magnify part of this photograph still another ten times, so that we have a total magnification of 1000, all resemblance to the original is lost. What we have now is a picture of the silver grain of the original photograph, and the grain structure is so coarse that we have lost the image structure (Fig. 4).

In the use of photography for many purposes, the granularity of the image is the controlling factor. The appearance of the granular structure is called *graininess*. If photographic materials could be made so as to be entirely free from graininess, the image when magnified would appear to be smooth, but we still could not see molecules by looking at photomicrographs of crystals, because in the second magnification we should be magnifying the structure of the microscopic image, and that is limited in its resolving power. A microscopic image, being produced by waves of light, has a resolving power limited by the size of the waves, and no advantage is gained by using magnification (either optical or photographic) beyond the limit set by the wave structure in the microscope.

Electron waves are much shorter than light waves, and an electron microscope can be used at a magnification of 100,000 diameters. Electron microscopes do show very large molecules.

The graininess of different photographic materials can be compared by examining them side by side at a constant magnification. One method of assigning a value of graininess to films would be to make a series of enlargements at different magnifications and then observe them from a constant distance to determine the maximum magnification at which the graininess is not objectionable. To do this is difficult, because the photographic work must be identical in making all the enlargements.

In 1920, Dr. L. A. Jones, in the Kodak Research Laboratories, designed an instrument for the measurement of graininess. It consists of a microscope system which projects a magnified image of the photographic deposit on a smooth, white screen. The screen is observed in a mirror at a distance from the eye of the observer that can be varied, and the mirror is moved so as to lengthen the distance until the granular structure of the deposit is no longer visible. The impression of graininess when viewing a photographic deposit in this way results not only from the grain size of the ultimate particles of silver, of which the image is composed, but also from the distribution and arrangement of these particles in clumps and, moreover, the further agglomeration of the clumps.

A good deal of work has been done in the Kodak Research Laboratories on the measurement of graininess. Colleagues of Jones in the Physics Department worked on the subject, notably A. C. Hardy and

E. M. Lowry. Their hope was always that a connection could be found between the appearance of graininess and the actual physical structure of the image; that is, its granularity.

If a photographic deposit is observed through a microscope and the light of a small area of the image is allowed to fall on a photoelectric cell, the intensity of the light varies as the deposit is scanned, and the

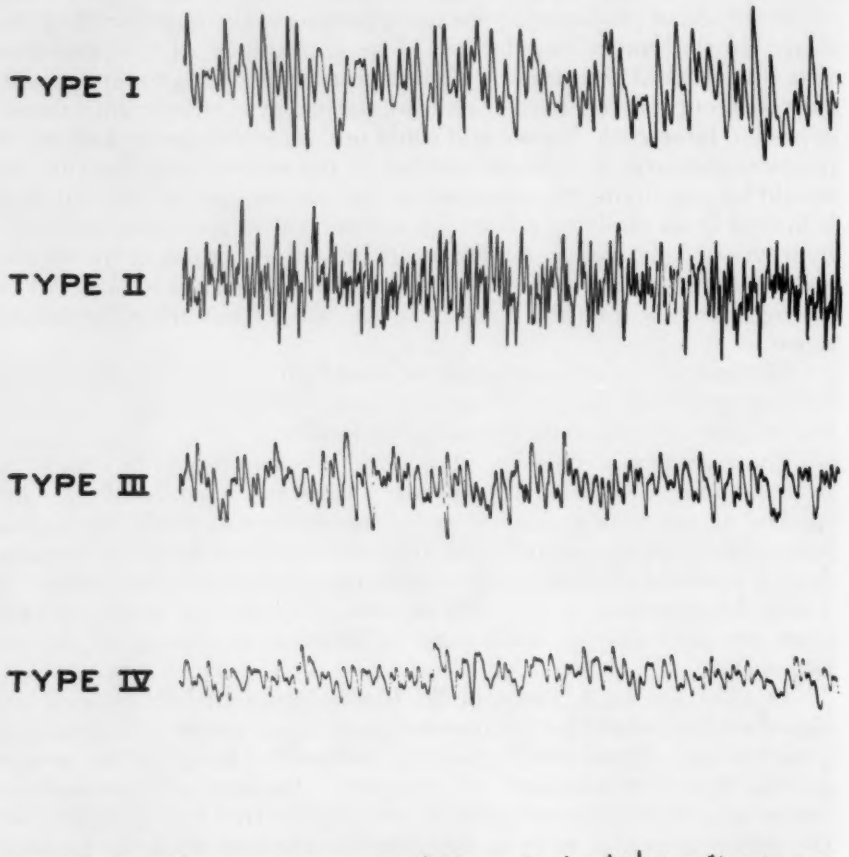

FIG. 5. Microdensitometer traces showing graininess of five typical emulsion types.

variation depends upon the granularity of the deposit. Thus, in Fig. 5, are shown the granularities of different kinds of astronomical plates as measured by a microdensitometer in which the light from a very small aperture traversing the image is allowed to fall on a photoelectric cell. The variations of density observed in this way are, of course, caused by

a variation in the size and relative location of the grains of the developed material. Very little work was done in this way on the measurement of granularity in the Kodak Laboratories until, in 1935, E. W. H. Selwyn, head of the Physics Department of the Kodak Laboratory in Harrow, England, published in the *Photographic Journal* an important paper on the theoretical distribution of the fluctuation of density in a photographic image and its dependence on the grain size. In 1939 Selwyn followed his theoretical paper by an experimental paper, in which he described the type of microdensitometer that he preferred to use for getting the granularity traces.

In the meantime, a number of workers in other laboratories, notably A. Goetz and W. O. Gould of the Ansco Laboratories, and A. vanKreveld, had been designing granularity instruments which were essentially special types of microphotometers, the methods of measuring granularity being based on the fluctuation of the density from the "average" density when very small isolated areas of the photographic image are observed successively.

This work on granularity attracted Jones' attention, and he concluded that it should be possible to establish the relations between measurements of granularity and determinations of graininess; that is, of the visual appearance of the photographic image under magnification. Jones was joined by G. C. Higgins in this work. The graininess of a number of samples was measured with the earlier Jones apparatus, and their granularity by various types of microdensitometers. Jones and Higgins concluded that none of the microdensitometer methods gave a result exactly parallel with the direct measurement of graininess, though it might be possible to assign to each material a value for granularity which would be proportional to the visual measurement of graininess. At least, it might be possible to arrange the materials in the same order by measurements of graininess and of granularity.

In later work, Jones and Higgins concluded that the perception of graininess is produced by the structure of the eye, and it is this which introduces the difference between granularity as observed with a microphotometer and graininess as derived from the observation of the magnified image. Instruments were constructed by Jones and Higgins to measure granularity in such a way that the scanning area would be related to the size of the retinal units in the eye, and when this was done measurements obtained physically were found to correspond to the visual observations of graininess.

When a photographic material is exposed to light with a knife edge in contact with the emulsion, the developed image does not end abruptly at the knife edge but encroaches into the shielded area. The light is diffused into the area shielded by the knife edge by refraction, reflection, diffraction, and scattering by the silver grains.

Photographic emulsion is, in fact, turbid, and the turbidity varies

with the kind of emulsion. High-speed coarse-grained emulsions are much more turbid than fine-grained emulsions. In 1909, I published a paper on the resolving power of photographic plates which involves the turbidity of the emulsion. I used a very simple system. A spectroscopic slit 1 cm. long and 1 mm. in diameter was covered by a small black wedge to give a range of intensities. The illuminated slit was photographed at a reduction of 25 diameters in a small and very rigid camera using a high-grade spectroscopic lens. The images on the plate were, of course, very small. The slit image was 4/10 mm. in height and about 1/25 mm. in diameter. The images were so small indeed that it was rather difficult to find them, and one of my assistants worked for several weeks, giving heavier and heavier exposures, until he finally realized that the images had been there all the time and were finally enormously overexposed! The appearance of these images of the slit is shown in Fig. 6. It is seen that at the point of most exposure the

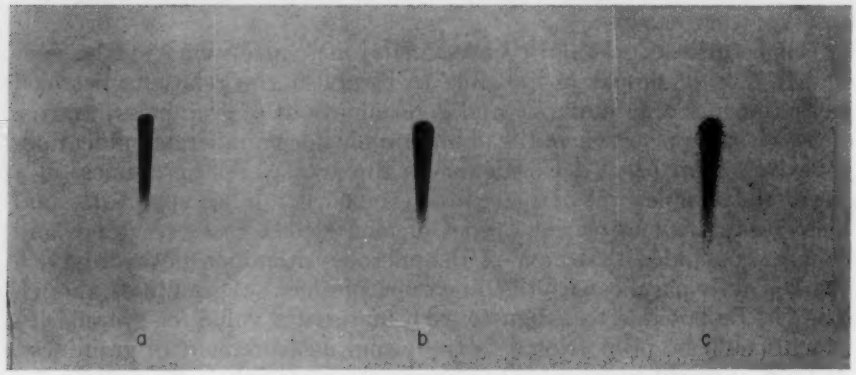

FIG. 6. "Tadpole" images used in measuring the resolving power of three typical emulsions.

image spreads sideways, whereas at the point of least exposure it narrows almost to a point, for which reason I called the images "tadpoles." In the figure are shown three different materials—microfilm, in which turbidity is kept to a minimum; a process film, whose turbidity is low but greater than that of microfilm; and a portrait film, which shows considerable turbidity and visible graininess in the magnified image. It is this spread of the image by turbidity which limits the resolving power of photographic emulsions.

The resolving power of an emulsion is its ability to record fine detail, measured by substituting a test object for the spectroscopic slit and black wedge used for making "tadpoles." The test object is photographed at such a reduction that some of the lines are not visible as separate structures. Many test objects have been used, but one that has been very satisfactory is shown in Fig. 7.

The numerical measure of the resolving power depends, of course, upon the quality of the lens used, and improvements have been made from time to time in lenses, each of which has resulted in a higher value being obtained for the resolving power of the materials tested. Resolving power also depends upon exposure and development, and the Kodak Laboratories have published many papers on the measurement of resolving power.

The sharpness of an image can be measured with the microdensitometer used for the measurement of granularity. It is necessary merely to

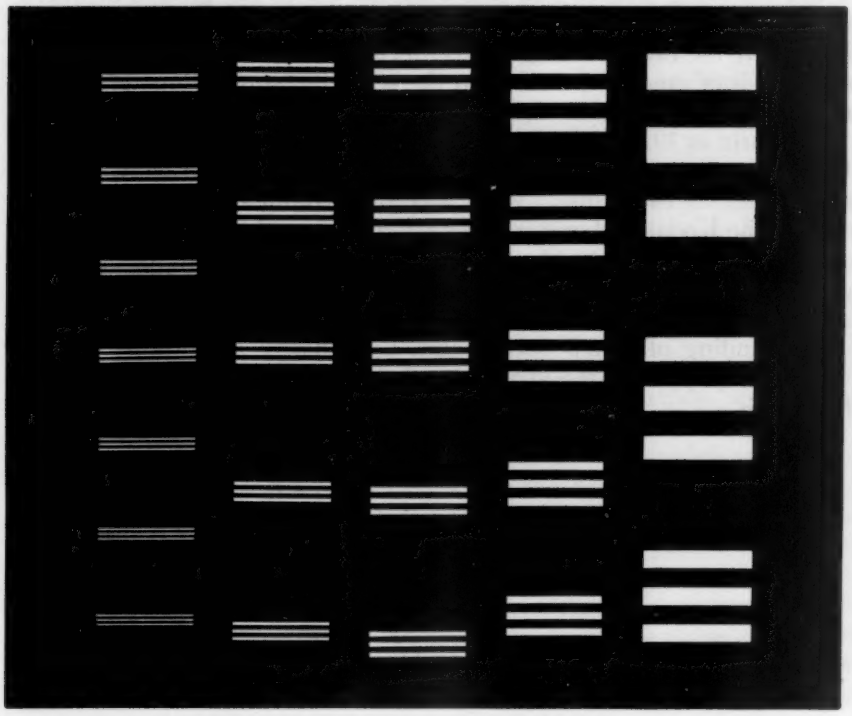

FIG. 7. Resolving power test object.

expose the material under a sharp edge and then scan the edge with the slit of the microdensitometer parallel to the edge of the image. Variation of the slope of the trace across the boundary between the exposed and unexposed areas gives a measure of sharpness. Higgins and R. N. Wolfe have studied the relation of the definition of a photograph to the sharpness and to the resolving power of the material. Definition is, of course, a composite effect, and since it is an impression made on the mind of an observer when he looks at a photograph, it can only be measured by the examination of a large number of photographs by a group

of observers. Different observers agree closely on the photographs showing the best definition. Higgins and Wolfe have shown that the definition depends upon both the resolving power and the sharpness, and they were able to work out a mathematical expression relating definition numerically to measurements of resolving power and sharpness.

Thus as the years have passed, work has continued on the detail observable in photographs and on the sharpness of the pictures obtainable. New materials have been made with greater sharpness and better resolving power. All this work has led to the growth of microphotography, as in the well-known microcopy systems which are so important for the recording of documents. Moreover, work on these subjects is of great importance in the scientific applications of photography, such as astronomy and microscopy.

As early as 1924, F. E. Ross wrote a monograph, "The Physics of the Developed Image," in which he summarized what was known of the subject up to that time. Ross was an astronomer who was on the staff of the Kodak Laboratories for some years and then returned to the observatories. He was therefore able to apply the results of photographic research to the needs of the astronomers. Higgins and Jones, on the other hand, were physicists specializing in optics, with a thorough understanding of visual optics, and, particularly as a result of their work, the physics of the structure of the image has been placed on a firm basis.

ON THE COMPLEX REPRESENTATION OF THE GENERAL EXTENSIONAL AND FLEXURAL PROBLEMS OF THIN PLATES AND THEIR ANALOGIES

BY

YI-YUAN YU [1]

ABSTRACT

The paper constitutes a generalization of the complex variable method developed by Muschelisvili and Lechnitzky for solving the extensional and flexural problems of thin plates to include, respectively, the effect of the body force and the lateral load. The two usual analogies between the two types of problems are also established in a quantitative manner for these general cases; and, as was observed by Southwell, the two analogies may be combined into one with the reversal of the sign of Poisson's ratio. As illustrations, several circular plate problems are solved by means of the generalized boundary equations derived, and the corresponding analogous problems of some of these are determined by means of the analogies.

As a necessity, a relation is established first between the indefinite integral and the line integral of a non-analytic function in the complex plane.

INTRODUCTION

In the present paper the general extensional and flexural problems of thin plates in which the body force and lateral load, respectively, are present will be formulated by means of the complex variable method developed by Muschelisvili and Lechnitzky. The method, as was first developed for extensional problems in two-dimensional elasticity theory, is described in detail in Muschelisvili's book (1)[2], an English translation of which is now available. Lechnitzky (2) further extended the method to flexural problems, and an English exposition of his results may be found in Sokolnikoff's notes (3). When the body force in an extensional problem is due to gravity, the compatibility equation remains homogeneous, no particular integral arises, and solutions have been obtained by the present author (4) for several such problems using a modified procedure of the complex variable method. In the general cases, however, due to the additional considerations of the body force and lateral load, the governing equations are no longer homogeneous, and particular integrals become involved. Though mentioned by the authors (1, p. 101; 3, p. 382), these particular integrals were not actually incorporated into the method. It is the purpose of this paper to carry out the details to incorporate these into the method so that problems involving the body force or lateral load may then be handled by the usual procedure. As examples, several circular plate problems will be solved on the basis of the boundary equations to be derived. The equations may also be applied to plates having shapes other than that

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

of a circle through the use of conformal transformation, the details of which will be the same as given by Muschelisvili in the original method and will not be presented here.

One reason for treating the extensional and flexural problems of thin plates together is that the usual analogies between the two types of problems, for the general cases in which the body force and lateral load are respectively present, may be revealed in a quantitative manner by a simple comparison of their boundary equations, once these are formulated by means of the complex variable method. The first of the two well-known analogies relates the first fundamental extensional problem in which boundary stresses are prescribed with the second fundamental flexural problem in which boundary deflections are prescribed. On the other hand, the second analogy relates the second fundamental extensional problem in which boundary displacements are prescribed with the first fundamental flexural problem in which the boundary moments and shearing force are prescribed. By virtue of the complex representation of the boundary equations, it may further be seen that, with the exception of a reversal of the sign of Poisson's ratio, both analogies hold simultaneously and therefore may be combined into one, as was also observed by Southwell (5). By means of these analogies, problems of one type may be converted into those of another. As illustrations, the analogous counterparts of some of the circular plate problems to be discussed will be sought.

In the course of the development the indefinite and line integrals of a non-analytic function will be used extensively. Since these are rarely discussed in the usual complex variable theory, a short account of these is desirable and will first be presented. To be of particular interest will be found the relation between these two types of integrals of a non-analytic function, which does not seem to have been discussed before.

NON-ANALYTIC FUNCTIONS

It is clear that any real function $F(x, y)$ when written in terms of the complex variable $z = x + iy$ will have to be also in terms of the conjugate variable $\bar{z} = x - iy$, that is,

$$F(x, y) = F(z, \bar{z}),$$

in which we have used the same notation F to denote the two functions, in spite of the fact that they are to be different in form. It follows that $F(z, \bar{z})$ thus deduced is not an analytic function of z.

We shall distinguish between the writing of indefinite integrals and that of line integrals. For example, referring to the above function F, we have the two types of integrals written as

$$\int F\,dz \quad \text{or} \quad \int F\,d\bar{z}$$

and

$$\int F\,(dx + idy) \quad \text{or} \quad \int F\,(dx - idy),$$

respectively.

The following relation between the two types of integrals of the above function F, which is to hold on any curve in the complex plane, will now be established:

$$\int F\,dz = \int F(dx + idy) + \int G(dx - idy) + C, \tag{1}$$

in which C is a constant, in general complex, and

$$G = \frac{\partial}{\partial \bar{z}} \int F\,dz.$$

The indefinite integral on the left side of (1) is evaluated at an arbitrary variable point on the curve, and the line integrals on the right side are integrated along the curve from any fixed point to the same variable point. The two integrals involving F would be equal to each other if F were an analytic function of z, for, in such a case, G would become zero. Since F is a real function and cannot be analytic in z, the two integrals are distinctly different from each other. As an example, we consider the function

$$F = x^2 + y^2 = z\bar{z}$$

and calculate the integrals with the unit circle as the curve, on which $z = \sigma = e^{i\theta}$. Thus,

$$\int F\,dz = \tfrac{1}{2}z^2\bar{z} = \tfrac{1}{2}\sigma,$$

$$\int F(dx + idy) = \int \sigma\bar{\sigma}d\sigma = \sigma,$$

$$\int G(dx - idy) = \int \tfrac{1}{2}\sigma^2 d\bar{\sigma} = -\tfrac{1}{2}\sigma.$$

The two integrals of F are seen to be different from each other, and the relation in (1) is satisfied with the possible exception of some constant term. To prove the relation for the general case we introduce the notation $P(z,\bar{z})$ to represent the indefinite integral $\int F\,dz$ and obtain its derivatives as

$$\frac{\partial P}{\partial z} = F, \quad \frac{\partial P}{\partial \bar{z}} = G.$$

The two integrals on the right side of (1) may now be evaluated along the curve as follows:

$$\int F(dx + idy) + \int G(dx - idy) = \int \frac{\partial P}{\partial z}(dx + idy) + \int \frac{\partial P}{\partial \bar{z}}(dx - idy) = \frac{1}{2}\int\left(\frac{\partial P}{\partial x} - i\frac{\partial P}{\partial y}\right)(dx + idy) + \frac{1}{2}\int\left(\frac{\partial P}{\partial x} + i\frac{\partial P}{\partial y}\right)(dx - idy) = \int\left(\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy\right) = P.$$

It is clear that an arbitrary constant may be added to this value of P which is evaluated at a certain variable point on the curve, and (1) is proved.

EXTENSIONAL PROBLEMS

Since thin plates will be dealt with, the state of generalized plane stress will be considered; and, needless to say, the state of plane strain may be similarly treated. If the usual body force potential V and Airy's stress function U are introduced so that the body force components in the x- and y-directions are given by

$$X = -\frac{\partial V}{\partial x}, \quad Y = -\frac{\partial V}{\partial y},$$

and the normal and shearing stresses given by

$$\tau_{xx} = \frac{\partial^2 U}{\partial y^2} + V, \ \tau_{yy} = \frac{\partial^2 U}{\partial x^2} + V, \ \tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y}, \tag{2}$$

the equilibrium equations will be satisfied and the compatibility equation becomes

$$\nabla^4 U = -(1 - \mu)\nabla^2 V, \tag{3}$$

in which μ is Poisson's ratio.

To formulate a relation between the displacement components u, v and the stress function, we substitute the stresses from (2) into the stress-strain relations and write the strains in terms of u, v; thus,

$$\left.\begin{aligned} E\frac{\partial u}{\partial x} &= \tau_{xx} - \mu\tau_{yy} = (1 + \mu)\frac{\partial^2 U}{\partial y^2} - \mu\nabla^2 U + (1 - \mu)V, \\ E\frac{\partial v}{\partial y} &= \tau_{yy} - \mu\tau_{xx} = (1 + \mu)\frac{\partial^2 U}{\partial x^2} - \mu\nabla^2 U + (1 - \mu)V, \\ E\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) &= 2(1 + \mu)\tau_{xy} = -2(1 + \mu)\frac{\partial^2 U}{\partial x \partial y}, \end{aligned}\right\} \tag{4}$$

in which E is Young's modulus. Introducing P so that

$$\nabla^2 U = P, \tag{5}$$

we may write (3) as

$$\nabla^2 P = -(1-\mu)\nabla^2 V.$$

It follows that

$$P = -(1-\mu)V + P_1, \tag{6}$$

in which $P_1(x,y)$ is a harmonic function. If Q_1 denotes the conjugate function of P_1, we may construct in the manner of Muschelisvili the following functions of the complex variable $z = x + iy$:

$$f(z) = P_1(x,y) + iQ_1(x,y),$$
$$\varphi(z) = \frac{1}{4}\int f(z)\,dz. \tag{7}$$

By making use of (5), (6), and (7) it may be shown that (4) leads to the following relation:

$$E(u+iv) = 4\varphi(z) - (1+\mu)\left(\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y}\right). \tag{8}$$

This result has the same form as that for the case in which the body force is absent.

We proceed next to solve the non-homogeneous equation (3). The complementary solution U_1 satisfies the equation $\nabla^4 U_1 = 0$ and has been shown by Muschelisvili to be expressible in terms of two analytic functions:

$$2U_1 = \bar{z}\varphi(z) + z\bar{\varphi}(\bar{z}) + \chi(z) + \bar{\chi}(\bar{z}),$$

in which $\varphi(z)$ is the same analytic function as defined in (7), $\chi(z)$ is the second analytic function, and overbars indicate conjugate quantities. This result had been obtained by E. Goursat,[3] whose formal approach may also be used here to determine the particular integral U_2. Since (3) may be written as

$$4\frac{\partial^4 U}{\partial z^2 \partial \bar{z}^2} = -(1-\mu)\frac{\partial^2 V}{\partial z \partial \bar{z}},$$

successive integrations yield immediately

$$U_2 = -\frac{1-\mu}{4}\int\int V\,dz\,d\bar{z}.$$

Arbitrary functions of z and $\bar{z}$ resulting from the various steps of integra-

[3] This was pointed out by Muschelisvili in his book (1, p. 111).

tion constitute precisely the complementary solution. The complete solution is therefore given by

$$2U = 2(U_1 + U_2) = \bar{z}\varphi(z) + z\bar{\varphi}(\bar{z}) + \chi(z) + \bar{\chi}(\bar{z}) - \frac{1-\mu}{2}\int\int V\,dz\,d\bar{z}. \quad (9)$$

By differentiation, we obtain

$$\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y} = \varphi(z) + z\bar{\varphi}'(\bar{z}) + \bar{\psi}(\bar{z}) - \frac{1-\mu}{2}\int V\,dz, \quad (10)$$

in which

$$\psi(z) = \chi'(z).$$

When the stress components given in (2) are substituted in the boundary conditions

$$X_r = \tau_{xx}\frac{dy}{ds} - \tau_{xy}\frac{dx}{ds}, \quad Y_r = \tau_{xy}\frac{dy}{ds} - \tau_{yy}\frac{dx}{ds},$$

in which X_r and Y_r are the components of the boundary force, and ds refers to an elementary length along the boundary, there results

$$X_r - V\frac{dy}{ds} = \frac{d}{ds}\left(\frac{\partial U}{\partial y}\right), \quad Y_r + V\frac{dx}{ds} = -\frac{d}{ds}\left(\frac{\partial U}{\partial x}\right).$$

The boundary value of $\dfrac{\partial U}{\partial x} + i\dfrac{\partial U}{\partial y}$ is therefore given by

$$\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y} = i\int (X_r + iY_r)ds - \int V(dx + i dy). \quad (11)$$

Substituting (10) into (8), we find the displacements in terms of $\varphi(z)$ and $\psi(z)$:

$$\frac{E}{1+\mu}(u + iv) = \frac{3-\mu}{1+\mu}\varphi(z) - z\bar{\varphi}'(\bar{z}) - \bar{\psi}(\bar{z}) + \frac{1-\mu}{2}\int V\,dz. \quad (12)$$

By using the same procedure given by Muschelisvili for the case in which the body force is absent, stresses may be shown to be given by

$$\left.\begin{aligned} \tau_{xx} + \tau_{yy} &= 2[\varphi'(z) + \bar{\varphi}'(\bar{z})] + (1+\mu)V, \\ \tau_{yy} - \tau_{xx} + 2i\tau_{xy} &= 2[\bar{z}\varphi''(z) + \psi'(z)] - (1-\mu)\frac{\partial}{\partial z}\int V\,d\bar{z}. \end{aligned}\right\} \quad (13)$$

Equations 12 and 13 were also obtained by Stevenson (6) without making use of Airy's stress function.

In the first fundamental boundary value problem of the extension of thin plates the boundary force (X_s, Y_s) is prescribed, which, according to (11) is equivalent to being given the value of $\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y}$ on the boundary. The boundary equation is therefore represented by (10). For the second fundamental extensional problem in which the displacements are prescribed at the boundary, (12) is the boundary equation. The functions $\varphi(z)$ and $\psi(z)$ which constitute the solution of a problem may be determined by means of these boundary equations.

As examples two problems related to a circular disk of radius R rotating about an axis through its center will be considered. If γ is the density of the material and ω the constant angular speed, the potential for the centrifugal force will be

$$V = k(x^2 + y^2) = kz\bar{z}, \quad \text{where } k = -\tfrac{1}{2}\gamma\omega^2. \tag{14}$$

The first problem now is the well-known one referring to the disk with a free boundary, for which (11) gives

$$\frac{\partial U}{\partial x} + i\frac{\partial U}{\partial y} = -\int V(dx + idy) = -R^3k\sigma = -R^2kz, \tag{15}$$

in which $R\sigma = Re^{i\theta}$ denotes the value of z on the boundary as given by the following identities:

$$z \equiv R\rho\sigma \equiv R\rho e^{i\theta},$$

the value of ρ being equal to unity for points on the boundary. Thus, according to (10), the boundary equation of the problem is

$$\varphi(z) + z\bar{\varphi}'(\bar{z}) + \bar{\psi}(\bar{z}) = -\frac{3+\mu}{4}R^2kz.$$

The analytic functions satisfying this equation are

$$\varphi(z) = -\frac{3+\mu}{8}R^2kz, \quad \psi(z) = 0, \tag{16}$$

which constitute the solution of the problem and were also obtained by Stevenson in a similar form.

Consider next the problem referring to the same rotating disk except that it is now enclosed inside a ring made of some much more rigid material. If the ring is assumed to be perfectly rigid and any rigid body rotation is disregarded, the displacements at the boundary

of the disk may be considered as zero, and (12) yields the following condition to be satisfied by the analytic functions at the boundary:

$$\frac{3-\mu}{1+\mu}\varphi(z) - z\bar{\varphi}'(\bar{z}) - \bar{\psi}(\bar{z}) = -\frac{1-\mu}{4}R^2kz.$$

The functions are therefore

$$\varphi(z) = -\frac{1+\mu}{8}R^2kz, \quad \psi(z) = 0. \tag{17}$$

The problem may also be considered as one in which the disk rotates within a perfectly smooth and rigid hollow casing.

If desired, stress and displacement components may be computed by the usual procedure of the method, that is, by substituting $\varphi(z)$ and $\psi(z)$ obtained for each problem, together with V from (14), into (12) and (13).

FLEXURAL PROBLEMS

We introduce the function Q into the equation that governs the transverse flexure of thin plates under lateral load:

$$\nabla^4 w = q/D = \nabla^2 Q,$$

in which w is the deflection of the plate, q the intensity of the lateral load, and D the flexural rigidity of the plate. Since this equation is of the same form as (3), we may write, similar to (9) and (10),

$$2w = \bar{z}\varphi(z) + z\bar{\varphi}(\bar{z}) + \chi(z) + \bar{\chi}(\bar{z}) + \frac{1}{2}\int\int Q\,dz\,d\bar{z}, \tag{18}$$

$$\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y} = \varphi(z) + z\bar{\varphi}'(\bar{z}) + \bar{\psi}(\bar{z}) + \frac{1}{2}\int Q\,dz. \tag{19}$$

The same notations $\varphi(z)$, $\chi(z)$, and $\psi(z)$ are used once more. The second derivatives of w may also be computed without difficulty; substituting the results into the well-known expressions for the bending moments M_x, M_y, twisting moment H_{xy}, and the shearing forces Q_x, Q_y,

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right), \quad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right),$$

$$H_{xy} = -D(1-\mu)\frac{\partial^2 w}{\partial x \partial y}, \quad Q_x = -D\frac{\partial}{\partial x}\nabla^2 w, \quad Q_y = -D\frac{\partial}{\partial y}\nabla^2 w,$$

we obtain

$$\left.\begin{aligned}
M_x &= -\frac{D(1+\mu)}{2}[2\varphi'(z)+2\bar{\varphi}'(\bar{z})+Q]-\frac{D(1-\mu)}{2}\\
&\quad\times\left[\bar{z}\varphi''(z)+z\bar{\varphi}''(\bar{z})+\psi'(z)+\bar{\psi}'(\bar{z})+\frac{L+\bar{L}}{2}\right],\\
M_y &= -\frac{D(1+\mu)}{2}[2\varphi'(z)+2\bar{\varphi}'(\bar{z})+Q]+\frac{D(1-\mu)}{2}\\
&\quad\times\left[\bar{z}\varphi''(z)+z\bar{\varphi}''(\bar{z})+\psi'(z)+\bar{\psi}'(\bar{z})+\frac{L+\bar{L}}{2}\right],\\
H_{xy} &= -i\frac{D(1-\mu)}{2}\left[\bar{z}\varphi''(z)-z\bar{\varphi}''(\bar{z})\right.\\
&\quad\left.+\psi'(z)-\bar{\psi}'(\bar{z})-\frac{L-\bar{L}}{2}\right],\\
Q_x &= -D\left[2\varphi''(z)+2\bar{\varphi}''(\bar{z})+\frac{\partial Q}{\partial x}\right],\\
Q_y &= -D\left[2i\varphi''(z)-2i\bar{\varphi}''(\bar{z})+\frac{\partial Q}{\partial y}\right],
\end{aligned}\right\} \tag{20}$$

in which

$$L=\frac{\partial}{\partial\bar{z}}\int Q\,dz,\quad \bar{L}=\frac{\partial}{\partial z}\int Q\,d\bar{z}.$$

These results may further be written in the following more compact form:

$$\left.\begin{aligned}
&M_x+M_y=-D(1+\mu)[2\varphi'(z)+2\bar{\varphi}'(\bar{z})+Q],\\
&M_y-M_x+2iH_{xy}=D(1-\mu)[2\bar{z}\varphi''(z)+2\psi'(z)+\bar{L}],\\
&Q_x-iQ_y=-2D\left[2\varphi''(z)+\frac{\partial Q}{\partial z}\right].
\end{aligned}\right\} \tag{21}$$

Equation 19 is obviously the boundary equation for the second fundamental boundary value problem in the flexural theory of plates, in which the deflection and slope of the plate are prescribed along the boundary. We still have to formulate the boundary equation for the first fundamental problem, in which the boundary moments and shearing force are prescribed. According to Lechnitzky (**2, 3**), the boundary condition for the latter case can be written as

$$\left[-M_\nu + i\int(Q_x dy - Q_y dx) - iH_{x\nu}\right]dx + \left[\int(Q_x dy - Q_y dx) + H_{x\nu} + iM_x\right]dy = \left[-m(s) + i\int p(s)ds + ic\right](dx - idy) \quad (22)$$

in which

$$m(s) = M_\nu \quad \text{and} \quad p(s) = Q_\nu + \frac{\partial H_{\nu s}}{\partial s}$$

are given functions of s along the boundary in terms of the prescribed boundary moments M_ν, $H_{\nu s}$ and shearing force Q_ν, and c is an integration constant. The letter ν denotes the direction normal to s. The integrals in (22) are evaluated along the boundary from some arbitrary point to another variable point. Substituting from (20) into (22) and integrating the result along the boundary once more in the same manner, we obtain

$$\frac{3+\mu}{1-\mu}\varphi(z) - z\bar{\varphi}'(\bar{z}) - \bar{\psi}(\bar{z}) = -\frac{1}{D(1-\mu)}\int\left[m(s) + i\int p(s)ds + ic\right](dx + idy) - \frac{i}{1-\mu}\int\int\frac{\partial Q}{\partial \nu}ds(dx + idy) - \frac{1+\mu}{2(1-\mu)} \times \int Q(dx + idy) + \frac{1}{2}\int L(dx - idy) + c_1, \quad (23)$$

in which $\dfrac{\partial Q}{\partial \nu}$ is the normal derivative of Q at the boundary and c_1 is another integration constant. By virtue of (1), in which F is now replaced by Q, (23) becomes

$$\frac{3+\mu}{1-\mu}\varphi(z) - z\bar{\varphi}'(\bar{z}) - \bar{\psi}(\bar{z}) - \frac{1}{2}\int Q dz = -\frac{1}{D(1-\mu)}\int\left[m(s) + i\int p(s)ds + ic\right](dx + idy) - \frac{i}{1-\mu}\int\int\frac{\partial Q}{\partial \nu}ds(dx + idy) - \frac{1}{1-\mu}\int Q(dx + idy) + c_1, \quad (24)$$

in which any additional constant due to c has been absorbed into c_1. By further writing the right side of (24) as $\dfrac{U + iV}{1-\mu}$, the boundary equation may also be written as

$$\frac{U + iV}{1-\mu} = \frac{3+\mu}{1-\mu}\varphi(z) - z\bar{\varphi}'(\bar{z}) - \bar{\psi}(\bar{z}) - \frac{1}{2}\int Q dz. \quad (25)$$

The boundary equation (19) for the second fundamental problem will now be applied to the problem of a clamped circular plate of radius R subjected to a uniform lateral load of an intensity q. Corresponding to the uniform load we have the function

$$Q = \frac{q}{4D} z\bar{z}. \tag{26}$$

Since the slope is zero in all directions at all points along the boundary, (19) becomes for this problem

$$\varphi(z) + z\bar{\varphi}'(\bar{z}) + \bar{\psi}(\bar{z}) = -\frac{1}{2}\int Q dz = -\frac{R^2 q}{16D} z,$$

which yields

$$\varphi(z) = -\frac{R^2 q}{32D} z, \quad \psi(z) = 0. \tag{27}$$

These represent the solution of the problem in the complex form.

As an example of the first fundamental flexural problem we consider another problem referring to the same uniformly loaded circular plate, now simply supported along its boundary. The boundary moments and shearing force are

$$M_r = H_{r\theta} = 0, \quad Q_r = -\frac{q\pi R^2}{2\pi R} = -\frac{Rq}{2},$$

from which

$$m(s) = 0, \quad p(s) = -\frac{Rq}{2}. \tag{28}$$

The function Q is the same as given in (26), and its normal derivative along the circumference of the plate is

$$\frac{\partial Q}{\partial \nu} = \left[\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial Q}{\partial y}\right)^2\right]^{\frac{1}{2}} = \frac{Rq}{2D}. \tag{29}$$

Since the plate is simply-connected, the constants c and c_1 in (24) are not essential and may be neglected. Substituting (26), (28), and (29) in (24) now yields

$$\frac{3+\mu}{1-\mu}\varphi(z) - z\bar{\varphi}'(\bar{z}) - \bar{\psi}(\bar{z}) = \frac{1}{2}\int \frac{q}{4D} z\bar{z} dz$$

$$-\frac{1}{1-\mu}\int \frac{q}{4D} z\bar{z}(dx + idy) = -\frac{3+\mu}{1-\mu}\frac{R^2 q}{16D} z,$$

from which

$$\varphi(z) = -\frac{3+\mu}{1+\mu}\frac{R^2 q}{32D} z, \quad \psi(z) = 0.$$

The problem is thus solved.

These two well-known problems of a circular plate were solved numerous times by previous authors using different methods and are presented here again mainly for the purpose of illustrating the method. The deflection, moments, and shearing force of the plate may be calculated easily by substituting $\varphi(z)$ and $\psi(z)$, together with Q, into (18) and (21). The results will be the same as previously obtained and will not be repeated.

ANALOGIES

Comparing the boundary equations (10) and (12) for the extensional problems with (19) and (25) for the flexural problems, we conclude that, for geometrically identical plates,

$$U \text{ corresponds with } w, \tag{30}$$

$$(1 - \mu)V \text{ corresponds with } -Q, \tag{31}$$

$$\mu \text{ corresponds with } -\mu, \text{ and} \tag{32}$$

$$Eu, Ev \text{ correspond with } \boldsymbol{U}, \boldsymbol{V}. \tag{33}$$

These were also observed by Southwell (5). Furthermore, the stresses in (13) for the extensional problem may be compared with the moments in (21) for the flexural problem; in fact, τ_{xx}, τ_{yy}, and τ_{xy} correspond respectively with

$$\frac{\mu M_x - M_y}{D(1 - \mu^2)} - \frac{Q}{1 - \mu}, \quad \frac{\mu M_y - M_x}{D(1 - \mu^2)} - \frac{Q}{1 - \mu}, \quad \frac{H_{xy}}{D(1 - \mu)}.$$

While there is a direct correspondence between the shearing stress and the twisting moment, there is none between the normal stresses and the bending moments. These relations between stresses and moments were not indicated by Southwell.

We shall now make use of these conclusions to find out the corresponding flexural problems analogous to the rotating disk problems which have been discussed. The corresponding lateral load potential has the form, according to (14) and (31),

$$Q = -k(1 - \mu)(x^2 + y^2),$$

from which the intensity of the lateral load

$$q = D\nabla^2 Q = -4Dk(1 - \mu)$$

is seen to be a constant, and k takes the value

$$k = -q/4D(1 - \mu). \tag{34}$$

Corresponding to the first extensional problem of a rotating disk with a free boundary, the boundary slopes of the plate in the flexural problem are given by, according to (15) and (30),

$$\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y} = -R^2kz = \frac{R^2q}{4D(1-\mu)}z,$$

from which

$$\frac{\partial w}{\partial s} = 0, \quad \frac{\partial w}{\partial \nu} = \frac{R^2q}{4D(1-\mu)}.$$

The plate is therefore clamped, with zero deflection and constant slope in the normal direction at the boundary. If the positive direction of q is downward, the curvature in the normal direction at the boundary will be concave downward. Since (25) is not referred to in the problem, (32) does not apply, and the analytic functions have the same form as given in (16). In terms of q the functions may also be written as

$$\varphi(z) = \frac{R^2q}{32D}\frac{3+\mu}{1-\mu}z, \quad \psi(z) = 0.$$

The analytic functions for the corresponding flexural problem analogous to the second rotating disk problem may similarly be obtained from (17) by putting k in terms of q and replacing μ by $-\mu$ according to (34) and (32) respectively; thus,

$$\varphi(z) = \frac{R^2q}{32D}\frac{1-\mu}{1+\mu}z, \quad \psi(z) = 0.$$

Relation (33) indicates that the quantities U and V on the boundary are equal to zero, that is,

$$\text{right side of (24)} = 0. \tag{35}$$

Since the region occupied by the plate is simply-connected, the constants c and c_1 in (24) may be taken equal to zero, as was pointed out before. Equation 35 therefore leads to the following relation to be satisfied at the boundary:

$$m(s) + i\int p(s)ds + i\int D\frac{\partial Q}{\partial \nu}ds + DQ = 0.$$

The bending moment and shearing force at the boundary are thus found to be

$$M_\nu = m(s) = -\frac{R^2q}{4}, \quad Q_\nu = p(s) = -\frac{Rq}{2},$$

in which we have assumed that $p(s)$ consists of shearing force only. The

total boundary shearing force may be seen to be equal to the total lateral load. While q is downward, the negative Q_r is upward, and the negative M_r tends to bend the plate concave downward along the boundary.

The combined analogy will be useful in the determination of analogous mixed boundary value problems of thin plates. This will be left for future study.

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THE PRESENT STATUS OF ATMOSPHERIC ELECTRICITY *

BY

W. F. G. SWANN [1]

INTRODUCTORY REMARKS

At the present time, my activities are concerned mainly with high voltage Van de Graaff generators, linear accelerators, and all the accompanying paraphernalia for torturing atomic nuclei. When not engaged with these, I have been living mentally in a galaxy, trying to pump up the energy of cosmic rays.

While thus engaged with my present professional family, Dr. Smith and Professor Holzer persuaded me to come here to renew acquaintance with one of my oldest sweethearts—atmospheric electricity. I will confess that I am delighted to find how she has blossomed during the nearly forty years in which I have not given her much attention. In fact, I find her more charming than ever and almost feel a kind of regret at having failed to take more advantage of my opportunities and at having forsaken her for so long.

When Professor Holzer asked me to give this speech, I asked him what kind of speech he wanted; for I realized that while I would be talking to men of eminence in the field, I would be talking to them after they had consumed a dinner. At such times, too much mathematical complexity is not in order. So I gave Professor Holzer three choices:

1. An ordinary after-dinner speech, primarily for the entertainment of people who have well-dined.
2. A semi-popular discourse emphasizing high spots in the atmospheric electric field, particularly those calling for further work, but without giving details or theoretical arguments, etc.
3. A regular technical address.

Professor Holzer chose No. 2, so that is what you are going to get.

And now I find myself in more trouble; for in the form of the speech chosen, there is little that I can say which will be new to anyone. All the worthwhile things have been said or will be said in the papers of yesterday, today and tomorrow. And so I have no new pictures of my

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own to show you. Apart from certain remarks pertaining to the origin of the earth's electric charge, all I have is your own pictures, and I can but arrange them in order. And so I appear before you, not as a painter of pictures, but as a picture hanger, and all that I can hope is that I shall hang your pictures in the settings in which you would like to see them.

GENERAL CONSIDERATIONS

The subject of atmospheric electricity has, in the past, concerned itself primarily with three sections: (1) The nature and origin of the atmospheric conductivity; (2) The nature and origin of the potential gradient, and the equivalent topic, the origin and maintenance of the earth's negative charge; (3) The origin and nature of precipitation phenomena, thunderstorms, etc. Finally, it used to be customary to include a fourth section under the title "The Penetrating Radiation." However, this last subject has outgrown all of its relations in the last three decades, and has taken unto itself another and more dignified title, so that, under this title, "The Cosmic Radiation," it stands by itself, concerned only with its own affairs and the cosmic phenomena of the universe. It lives in a state of affluence supported by much money, and consorts only with the aristocrats of physics—the nuclear physicists. We poor atmospheric electric physicists can hardly claim it any more as a relative. We must be content with one another and our old friends, the meteorologists, whose popular representative is the weather man, for after all, the princes of cosmic rays give very little to our science. It is true that out of their abundance they seem to support the conductivity over the ocean, but to the conductivity on land they only make a token payment. It must be confessed, however, that we, the atmospheric electric physicists, give the cosmic ray people very little from our own science.

I have been advised to keep this speech on the non-technical level, and the natural limitations of time prevent my doing more than touching upon certain topics. Apart from a few remarks on measurements in general, I must direct attention to the situation as regards atmospheric conductivity and must make my main concern the origin and maintenance of the earth's charge.

To refer briefly to a matter having to do with measurements, one is conscious, particularly in the early history of the subject, of many things masquerading as fundamental phenomena but which, as a matter of fact, are, in the last analysis, merely the natural consequences of the operation of the laws of electrostatics. Thus, for example, neglecting the motion of electricity produced by air currents, we may say that the fact that the conductivity increases with altitude automatically results in the existence of a potential gradient which diminishes with altitude and this in turn, by the fundamental laws of electrostatics, results in the existence in the atmosphere, below the point at which the potential

gradient is small, of a positive charge equal and opposite to the charge upon the surface of the earth. The increase of conductivity with altitude is itself a primary phenomenon depending upon such factors as the distribution of radioactive material in the atmosphere, the variation of mobility of ions with pressure, and so forth. However, with this variation of conductivity established, the atmospheric charge distribution and the potential gradient variation follow as an inevitable consequence, and if we started with a condition in which there were no potential variation and no positive charge, the flow of electricity itself would result in the appearance of a positive charge and of a variation of potential gradient with altitude.

Even today, a word of caution is necessary to the effect that, in appraising the relative significance of atmospheric electric phenomena, it is important that we should separate things which are more or less independent from those which are directly related by the known laws of electricity and magnetism. Intimately related to this matter is that of the significance of instrumental measurements. Frequently the relation of that which is measured to the fundamental quantity which it was the intention to measure is not free from uncertainty.

Thus, for example, a hemispherical mound a mile in diameter and half a mile high might seem, superficially, to present a reasonably flat surface at the top, for potential gradient measurements. However, since the field on the top of such a hemisphere, regardless of its dimensions, is three times the field over a flat surface obtained when the hemisphere is removed, it is easily seen that serious errors may result from neglect of the inevitable implications of the fundamental electrostatic laws which govern this matter. Lack of sufficient care in a correct appraisal of the effects of the atmospheric potential gradient in inducing charge on parts of the measuring apparatus has, in the past, led to serious errors in the results obtained, errors which have sometimes masqueraded as fundamental phenomena of nature calling apparently for erudite discussions as to their causes.

Atmospheric electricity is not the only science in which this kind of thing has happened. The study of electrets has revealed phenomena which seem at first sight puzzling, and indeed, have been considered by some to be inconsistent with the laws of electrostatics, only to find that all is explained in a simple manner when the implications of these laws are traced.

Another matter which has hindered the interpretation of atmospheric electric data has been the sensitivity of such data to local phenomena of such a nature that, through their complexity, it is difficult to define the extent of their influence. Dust, smoke, sand, etc., have often been responsible for yielding results which are of very little value as regards their interpretable significance.

Fortunately, the great oceans have provided us with a territory more uniform in their characteristics and freer from undefinable local agencies

than is the case with land territories; and so it has come to pass that ocean observations have tended to give us a clearer picture of the essentials. Also, to some extent, observations at high altitudes, in spite of their difficulty, are susceptible of cleaner interpretation than are those at the ordinary ground observatory.

Atmospheric Conductivity

Atmospheric conductivity results from a balance between rate of production of ions by ionizing sources and rate of dissipation of ions by recombination with one another or with dust or other nuclei. With a wide degree of generality, we may assume that the time rate of change of the number n of ions per cc. of one sign is related to the rate of production q per cc. by the equation

$$\frac{dn}{dt} = q - \alpha n^2 - \beta n.$$

The βn term is frequently neglected in ordinary laboratory phenomena having to do with artificially produced ionization, but in atmospheric electricity it plays a fundamental role. It is natural to attribute this term to the presence of dust nuclei and the like. The matter is not as simple as might at first appear, however; for even after such nuclei have been removed as far as it seems feasible to remove them, there yet seems a limit beyond which β cannot be further reduced. Many years ago, I played with the idea that the βn term might arise from columnar ionization along the tracks of the ionizing particle, and indeed this idea has been developed by others since that time. However, I was never able to convince myself of the harmonization of columnar ionization with the phenomena observed in the case where the ionizing particles are electrons. I believe that the significance of the βn term calls for more study even at the present time.

The extent to which different ionizing agencies contribute to the conductivity has always been a subject of primary interest. According to Prof. Victor F. Hess, the following table represents the contribution to q—the number of ions produced per cc. per second—by the various agencies referred to over land. The following table, due to Hess, is

Production of Ions Near Earth's Surface
Over Land in Ion-Pairs/cc/sec.

	Ionizing Rays			
Ionizer	α	β	γ	Total
Radioactive Matter:				
In earth	0	0.1	3.0	3.1
In air	4.6	0.2	0.15	4.9
Cosmic Radiation	—	—	—	1.5
Total				9.5

interesting as indicating the variation of the rate of production of ions by the various agencies, with distance from the ground over the relatively small range of one meter. Naturally, such data may be expected to be highly local in character.

Production of Ion Pairs near Earth's Surface

Distance from Surface	3 cms.	100 cms.
Ion Pairs:		
α rays	3.58	1.76
β rays	2.18	0.47
γ rays	3.76	3.21
Cosmic rays	1.96	1.96

Over the great oceans, ionization is primarily limited to the cosmic ray contribution, so that the rate of production of ions over land is six times as great as the corresponding value over the sea. The fact that the number of ions per cc over the ocean is fully half as great as over land calls for a smaller value of β for the ocean than for the land. However, the relative magnitude of β necessary to harmonize the land and sea ionization is one which seems reasonable in the light of the possibilities.

Atmospheric Contamination by Atomic Explosions

In view of the relatively small ion content of the atmosphere, and of the large amount of radioactive material released in an atomic explosion, it is interesting to inquire the extent to which such an explosion can materially affect atmospheric electric observations.

The whole of the atmosphere over the earth contains normally about 6×10^7 curies of radium emanation and this is responsible for most of the ionization. Now, an explosion of a bomb of the Hiroshima type puts into the atmosphere an amount of radioactive material which, measured in curies, is about three times this amount 24 hours after the explosion. One week after the explosion it is about one-fifth of this amount. A year after the explosion it is about 1/600th of this amount.

Now, of course, all of this radioactive material does not distribute itself uniformly immediately. It probably goes along in very concentrated form following the direction of the air currents. There seems to be a considerable amount of misunderstanding about this matter. I think, however, that anyone who has watched the dissipation of smoke from an airplane writing in the sky would have no difficulty in believing that if one released 100 captive balloons at an altitude of a few thousand feet, he would not find them wandering off in all directions hither and yon, but would observe that they traveled along more or less together in such fashion that even after a journey of many miles one might expect to find them more or less clustered together or, at any rate, not very widely separated. It is to be anticipated that the radioactive material will

travel along in very much the same fashion, so that whether or not it will constitute a hazard, will depend upon where one happens to be in relation to the wind currents and, of course, upon precipitation; for if there is no precipitation, the rays from the radioactive material will be absorbed to a large extent before reaching the earth.

Most of the radioactive products from fission bombs have a short life and are not likely to contaminate the atmosphere for very long. However, in the case of the hydrogen bomb we are concerned with tritium, which has a half life of 10,000 years. What becomes fixed, however, in building a hydrogen bomb of assigned energy yield is a certain minimum *number* of tritium atoms. If the process is, let us say, 10 per cent efficient, then 10 per cent of these atoms will be pumped into the atmosphere. However, the very fact that the half life is so long—10,000 years—means that the fraction of the total number of atoms which break up per second—and so the contaminatory effect on our experiments—is not necessarily large. In addition to this, the range of penetration of the radiation from tritium is very small. The effect lasts a long time, but physicists do not usually do experiments which last 10,000 years—only astronomers do that, and they do it in contemplation.

Of course, contamination over the sea tends to be rapidly disposed of by the products of contamination becoming spread throughout the volume of the ocean waters, so that their radiations cannot get through to the surface. Radioactive material deposited on land can be a more serious matter. Presumably, however, much of it would get into the air and in process of time this would disappear aloft or become dissolved in the oceans.

THE ORIGIN AND MAINTENANCE OF THE EARTH'S CHARGE

I now turn to the all-important question of the earth's charge and its maintenance. The average value of the potential gradient at sea level is of the order 120 volts per meter. The potential gradient varies considerably over land, but over the great oceans it is sensibly constant, and its average surface value is about 126 volts per meter.

The conductivity of the atmosphere increases with altitude by a factor of 10 for an altitude change of 7000 meters, and in this conference we have learned of data continuing the increase up to much higher altitudes. The altitude increase of conductivity leads, through direct application of the equation of continuity and the laws of electrostatics, to the conclusion that there should be a positive charge in the atmosphere and that the potential gradient should decrease with altitude. Both of these facts are confirmed by experiment, and the potential gradient at an altitude of 10 kilometers is only about 2 per cent of the sea-level value.

The average value of the surface current density is about 3×10^{-16} amp/cm^2/sec. The total negative conduction current from the whole

earth is about 1800 amperes. The rate of conduction of electricity into and through the atmosphere is sufficient to insure that 90 per cent of the earth's charge would disappear in about 10 minutes if there were no means of replenishing the loss.

Theories of the maintenance of the earth's charge have divided themselves into two main categories—those in which the whole mechanism of replenishment is to be found within the atmosphere itself, or within the earth, and those in which it is supposed that negatively charged particles are shot from outside through our atmosphere to the earth.

A third type of theory has envisaged more drastic hypotheses according to which, by a very slight modification of the fundamental laws of electrodynamics, a continual very slow rate of death of positive electricity is maintained as a result of the earth's rotation—a death amounting to 1 proton per cc. per day. This would result in building up a continually increasing surplus of negative electricity were it not for the conduction current which carries it off into space.

Prominent among the first named category of theories are those involving precipitation, in which it is supposed that through the agency of rain or other forms of precipitation, or through associated lightning flashes, negative charge is brought down to the earth against the influence of the electric field, builds up thereon, and finally flows back into and through the atmosphere in the form of the atmospheric electric conduction current.

A somewhat different form of mechanism, proposed by Ebert, invoked the fact that ionized air, emerging from the pores of the earth's surface during periods of falling barometric pressure, comes into the atmosphere with a net positive charge on account of the diffusion of the negative ions to the walls of the pores. The positive ions were supposed to be carried upwards by rising air currents. The net negative charge on the earth's surface then built up a field which resulted in the usual negative conduction current which passed into the air, eventually to join with and annul the positive current arising as aforesaid.

Precipitation theories were the first to claim attention. Such theories experienced obstacles prominent among which was the fact that precipitation brought down charges of both signs and, while the negative precipitation was 10 times as large as was necessary to maintain the earth's charge, there was an almost equal precipitation of positive charge, and the evidence of earlier days seemed to indicate that the excess was in favor of the positive sign, that is, of the wrong sign for purposes of maintaining the earth's charge.

Another obstacle arose from the fact that precipitation theories resulting, as they do, in a separation of charges, with one sign remaining temporarily in the atmosphere while the other sign is deposited on the earth, lead to a condition in which the opposite charges remain bound to each other's vicinities. The negative density on the earth is held bound to the positive in the atmosphere, and no surface charge or po-

tential gradient is therefore to be expected outside of the region where precipitation occurs. This difficulty, which also affected the Ebert theory, became alleviated when the effect of the high conductivity of the upper atmosphere became recognized. We have now, in fact, to recognize, to a first approximation, two spherical conductors, the earth's surface, E, and the so-called conducting layer, S. The displacement of charges in the atmosphere, or from atmosphere to ground, sets up a difference of potential between these layers, a difference of potential which is handed around to all parts of the earth.

Thus, some of the earlier difficulties confronting precipitation theories evaporated in the light of fuller considerations born of the existence of the conducting layer; however, during the period in which it seemed that these difficulties might be significant, it became customary to seek for more drastic means of replenishment in the form of a compensating current shot into the earth from outer space. At the time when such a proposal was made by G. C. Simpson, one shuddered at the thought of the particle energies necessary to provide penetration through the atmosphere. However, the development of our knowledge of cosmic rays soon taught us that this matter represented the least of all obstacles to the acceptance of corpuscular replenishment. We are accustomed now to deal in cosmic ray particles with energies hundreds or thousands of times the value necessary for penetration of the atmosphere. However, by the same token, our greater knowledge of these particles has placed greater limitations upon the use which we may permit ourselves to make of them. We now know the actual number of such particles entering the earth. It is 100,000 times too small to present any possibility of using the cosmic rays as a source of replenishment of the earth's charge, and we have very little reason to believe that we have missed a whole category of charged particles. The only possibility in this direction lies in the supposition that particles of sufficiently high energy would be undetectable; and while there is some theoretical reason to believe that this may be so, the necessity for invoking such a drastic hypothesis has passed. Superposed upon all this is the fact that if we had a corpuscular current sufficient in amount to replenish the loss of the earth' charge, the rate of production of ions which it would be expected to produce in the atmosphere would be a hundred thousand times that known to exist. I will not deny that it might be possible to doctor up the corpuscular hypothesis to meet the primary objections confronting it. However, in the light of the clarification of the whole picture as represented by precipitation theories, the development of such theories to the point of eliminating the old difficulties which faced them, and the fact that they permit an understanding of diurnal variation phenomena in a manner which, as we shall see, would present a very great problem for any theory based on an incoming corpuscular current, renders it undesirable to pursue the corpuscular hypothesis further in this epoch.

Similar remarks of obsoleteness apply to theories concerned with modification of electrodynamic laws to permit a death of positive electricity as a result of the earth's rotation, with the accompanying build up of the negative counterpart to supply material for the atmospheric electric conduction current.

Having, therefore, discussed briefly the past history of the subject, let us take a fresh breath and inquire how things stand today. In speculating upon the possibilities, two things emerge as of outstanding significance.

The first matter of importance is the existence of a conducting layer, S, which, with the earth's surface, E, constitute two spheres, each sphere being at a potential which is the same at all parts but which, in the case of the conducting layer, may vary with time. It is not without interest to observe that if we were to draw these two spheres on a piece of paper, it would be difficult to draw a line so thin that its thickness was not too great to represent the distance between them.

The second matter of outstanding significance is S. J. Mauchly's discovery to the effect that the diurnal variation of the potential gradient follows universal rather than local time over the oceans. This means that a part from local effects, the maximum of the potential gradient always occurs at the same instant at all places on the earth's surface. It is not without interest to inquire as to the longitude for which this instant is local noon. It so happens that the longitude in question is the longitude of some place in California.

The dependence of the potential gradient upon universal time is something which it is almost impossible to account for on the basis of an incoming corpuscular current of constant intensity coming from some place external to the earth. Such an incoming corpuscular current of constant magnitude could only be consistent with a diurnal variation by the existence of a distortion of the conducting layer, and a distortion which followed universal time. Thus, for example, if the conducting layer were depressed downwards at some place, the potential gradient would become increased, since the layer is at the same potential at all places. Such phenomena as atmospheric tides would lead only to potential gradient variations which followed local time. The only kind of what I may call pseudo-tide phenomena which could give a potential gradient following universal time in the light of the existence of a constant total supply current is one where the radius of the conducting layer increased periodically with time and equally at all places, and such a motion is not consistent with any known cause. Even if it existed, there is no known cause which could determine its phase. Some years ago, I made a suggestion to the effect that when the sun was overhead at any place, the layer would be depressed at that place, resulting in an increase of potential gradient. Here again, however, we have a phenomenon which would follow local time.

A possibility intimately related to this idea is the following. Sup-

pose the electrical resistance per unit area between the conducting layer and the earth varies with time, and to different extents in different places. Then, since the potential of the conducting layer is the same at all places at any instant, the conduction current based upon a constant total current will vary from place to place in inverse proportion to the resistance per unit area. If the resistance variations referred to are confined to the higher altitudes, the potential gradient at sea level will vary proportionally to the current density and so inversely proportional to the columnar resistance per unit area. Again, in the normal course of events, we anticipate a variation of columnar resistance, and so of potential gradient, which follows local time. However, by a rather far-fetched hypothesis, we can secure conditions in which a component of the variation follows universal time. Thus, suppose the alteration in columnar resistance follows local time in a general way but that the *extent* of the variation depends upon local conditions. To take an exaggerated case, suppose that, at a certain longitude, the resistance variation was much greater than anywhere else—possibly on account of local meteorological conditions—then, superposed upon the general local time variations, we should have a variation following universal time.

It must be admitted that such avenues of escape from the difficulty such as I have here envisaged are very speculative. When, however, we give up the concept of a constant corpuscular current and seek replenishment from the atmosphere itself, matters assume a much more natural role. The basic motivating cause which we now seek is one where the supply current varies with time, and so determines a conduction current and a potential gradient varying with time.

Thus, to take an exaggerated case, suppose we have replenishment as a result of thunderstorms occurring at one place on the earth's surface and that the thunderstorm frequency at that place follows local time. Then the supply current will follow what is local time for that place, and so universal time for the earth as a whole.

In the next simplest approach to a picture, we may imagine a condition in which thunderstorm activity could be divided into two components, a component which was distributed uniformly over the earth's surface, and a component of larger value contributed by the specialized region. The first of these components would be expected to result in a potential gradient which showed no diurnal variation, while the second would result in a diurnal variation following universal time. As regards the second, we should have a maximum of potential gradient all over the earth at the instant when, in the specialized region, there was a maximum of thunderstorm activity. If, 12 hours later, there were a minimum of thunderstorm activity at the specialized place, then at that time we should have a minimum of potential gradient all over the earth.

If there are two places where the thunderstorm activity has pronounced maxima and minima 12 hours apart, then we shall still get a

net potential gradient variation which follows universal time. Of course, if all places are equally favored as regards thunderstorms, the potential gradient will show no time variation at all.

We could secure a 12 hour period by supposing conditions in which, at the specialized place, there was a 12 hour period in thunderstorm activity. However, the fact that the 12 hour period seems to be absent over the oceans, and at the top of the Eiffel Tower, for example, where we are well removed from the complicated meteorological conditions associated with the earth's surface, these facts invite the supposition that no more complicated assumption than that of the 24 hour period in the thunderstorm activity is necessary to account for the facts.

Now, that the basic ideas underlying the foregoing considerations are in line with the facts is borne out by the fact that the total thunderstorm activity does follow roughly a 24 hour period in universal time, and by the further fact that the time of maximum thunderstorm activity corresponds to the time of maximum potential gradient.

I have already referred to the fact that maximum potential gradient occurs at a time when it is noon in some portion of California. I do not wish to attach too much significance to this matter. However, it is evident that an important field of activity is to be found in the correlation of thunderstorm activity with atmospheric electric data, with special attention to the frequency of occurrence of thunderstorms at different parts of the earth's surface, and to the seasonal and similar variations of thunderstorm activity in different places.

The situation presented by a thunderstorm is, I believe, typically one roughly represented by an electric doublet in the atmosphere, the positive charge being on the top and the negative charge below. For complete electrostatic considerations we have to envisage the images of this doublet in the earth below and in the conducting layer above. The net result is that we have negative electricity streaming down to the earth below and positive electricity streaming up to the conducting layer, where it spreads out, as does the negative charge on the earth, to the extent necessary to insure constancy of potential for each sphere. The potential difference thus established between the two spheres then sets up the atmospheric-electric conduction current.

A lightning discharge at any place on the earth's surface might well be detectable at any other part of the surface. This, a discharge of 30 coulombs would change the potential difference between the earth and the conducting layer by δV, where

$$\delta V = \frac{30 \times 3 \times 10^9 (4\pi h)}{4\pi r^2} \times 300 \text{ volts},$$

where h is the height of the conducting layer, and r is the earth's radius. Putting $h \sim 50 \times 10^5$; $r = 6.5 \times 10^8$,

$$\delta V \sim \frac{30 \times 3 \times 10^9 \times 50 \times 10^5 \times 300}{4 \times 10^{17}} \sim \tfrac{1}{3} \times 10^8 \text{ volts}.$$

Even if this were distributed sensibly equally throughout 10 kilometers, it would amount to 0.0003 volt per centimeter. Such a voltage should be detectable by radio methods, but in view of the large number of thunderstorms occurring per second, the problem of disentangling the separate storms presents a very serious problem.

The thought that atmospheric electric phenomena find their origin in the atmosphere itself naturally invites an attempt to balance the various sources of contribution and see whether they add up to zero. Data to this effect have been obtained by Dr. Wormel, who finds, for supply of positive electricity to the earth, the following contributions from four sources: the ordinary atmospheric electric current, precipitation, point discharge below clouds, and lightning.

Conduction current	60 Coulombs per km^2 per year.
Precipitation	20 Coulombs per km^2 per year.
Point discharge	−100 Coulombs per km^2 per year.
Lightning	− 20 Coulombs per km^2 per year.

These do not add up to zero but to −40 Coulombs/km^2/yr. However, the average data for the oceans seem to reveal a conduction current nearly twice that assumed by Wormel, and I believe that the consensus of opinion is that the large point discharge current is not really representative of the earth as a whole—including the oceans. We are fortunate in having Dr. Wormel with us, and doubtless it will be possible to iron this matter out.

Conscious of the fact that the thunderstorm contribution to the earth must be the negative of the thunderstorm contribution to the conducting layer (since the total contribution must be zero) and recognizing that it is easier to make measurements over a thunder cloud than beneath it, Drs. Gish and Wait, by measuring the conductivity and potential gradient over thunderclouds, have sought to ascertain whether, indeed, the total current from all thunderstorms can provide a balance to the conduction current. They have, in fact, asked themselves the question of how many thunderstorms must be going on at any one time all over the earth in order to supply to the conducting layer a current equal and opposite to the atmospheric electric current.

As a result of measurements on some 21 storms, they arrive at a figure between 0.5 amp. and 0.8 amp. as a typical thunderstorm current. Taking 1800 amps. as the total conduction current, the number of thunderstorms per second necessary to compensate the conduction current turns out to be 2200 if we take the larger current (0.8 amp.) and 3600 if we take the smaller value (0.5 amp.). These values for the number of storms are greater than those sometimes quoted—values of the order 1800. However, a consideration of all the circumstances makes it not unreasonable to contemplate a value as high as 3600. At any rate, these considerations serve to illustrate the necessity of securing more data in this very important field.

ON THE AMPÈRE FORCE

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

1. INTRODUCTION

In a previous paper (1),[3] we studied possible equations for the force between unaccelerated charges. An attempt was made to prove that the only equation consistent with the accepted ideas of metallic conduction and the Ampère postulates I–IV is the equation originally stated by Gauss:

$$\mathbf{F} = -\,\mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left[1 - \frac{3}{2}\cos^2\theta\right]. \tag{1}$$

The validity of our proof has been questioned by Dr. F. W. Warburton (2), who has discovered an error in sign in one term of our Eq. 10. As Dr. Warburton rightly points out, the correction of this error allows the introduction of an arbitrary constant C and this destroys the uniqueness of Eq. 1. A similar difficulty was found by Warburton in his own formulation of electrodynamics (3). The theory of Ritz (4) also contained an arbitrary constant which has never been evaluated.

The present paper is a re-examination of the Ampère force, to determine under what circumstances C enters the equations and what experiments could be devised to evaluate C. Strangely enough, C disappears in most practical equations dealing with force and induced emf. And in some of the remaining equations, the terms containing C are found to contradict experiment. Thus it is necessary to set $C = 0$, which leaves all the equations of the new electrodynamics exactly as they were (5).

We conclude, therefore, that though Warburton's criticism is justified, it has no practical significance. This paper deals with the Ampère force: the other two terms of the general equation of electrodynamics (5) are not affected by Warburton's criticism and will not be considered here. We are particularly interested in two questions:

(*a*) What is the evidence that $C = 0$?
(*b*) How do the predictions of the new electrodynamics compare with classical results?

These questions will be considered in the following pages.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

2. THE SIN θ COS θ-TERM

Our previous Eq. 10, expressing the Ampère force between current elements, should read

$$d^2\mathbf{F}_2 = \frac{N_1N_2A_1A_2|Q_e|^2ds_1ds_2}{r^2}\{\mathbf{a}_r[f_r(v_1,\theta_1) + f_r(v_2,\theta_2+\pi) - f_r(v_3,\theta_3)] + \mathbf{a}_n[f_y(v_1,\theta_1)\cos\eta + f_y(v_2,\theta_2+\pi) - f_y(v_3,\theta_3)\cos\eta_3 - f_x(v_1,\theta_1)\sin\eta + f_x(v_3,\theta_3)\sin\eta_3] + \mathbf{a}_p[f_x(v_1,\theta_1)\cos\eta + f_x(v_2,\theta_2+\pi) - f_x(v_3,\theta_3)\cos\eta_3 + f_y(v_1,\theta_1)\sin\eta - f_y(v_3,\theta_3)\sin\eta_3]\}, \quad (2)$$

where the notation is evident from Fig. 1. The two current elements are I_1ds_1 and I_2ds_2 having cross-sectional areas A_1 and A_2. The num-

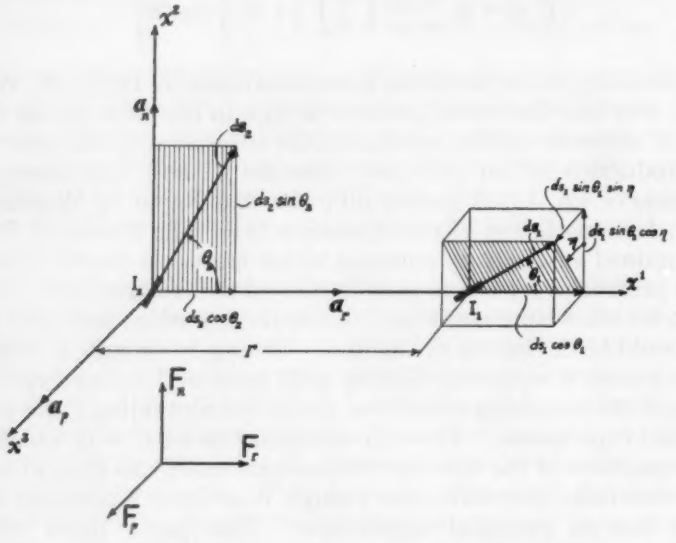

FIG. 1. Current elements.

bers of free electrons per unit volume are N_1 and N_2, and the magnitude of the electronic charge is $|Q_e|$. Magnitudes of electron velocity in the two conductors are v_1 and v_2 with respect to the element I_1ds_1, and

$$v_3^2 = v_1^1 + v_2^2 - 2v_1v_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos\eta),$$

$$\cos\theta_3 = \frac{1}{v_3}(v_1\cos\theta_1 - v_2\cos\theta_2).$$

The f's are functions to be evaluated.

As originally printed (1), the penultimate term of Eq. 2 had a nega-

tive sign instead of a positive sign. The most general equation for force between charges, obtainable with the old equation, was

$$f_r(v,\theta) = \frac{1}{4\pi\epsilon}\left(\frac{v}{c}\right)^2 A \sin^2\theta + B\cos^2\theta], \quad f_y(v,\theta) = 0, \quad f_z(v,\theta) = 0,$$

which led uniquely to the Gauss equation when the Ampère postulates were introduced.

But as Warburton pointed out (2), the correct sign allows the use of a $\sin\theta\cos\theta$-term in f_y, giving

$$\begin{cases} f_r(v,\theta) = \dfrac{1}{4\pi\epsilon}\left(\dfrac{v}{c}\right)^2 [A\sin^2\theta + B\cos^2\theta], \\ f_y(v,\theta) = \dfrac{1}{4\pi\epsilon}\left(\dfrac{v}{c}\right)^2 C\sin\theta\cos\theta, \\ f_z(v,\theta) = 0. \end{cases}$$

Substitution of these f's into Eq. 2 above yields

$$d^2\mathbf{F}_2 = \frac{|I_1||I_2|ds_1ds_2}{4\pi\epsilon c^2 r^2}\{\mathbf{a}_r 2[A\sin\theta_1\sin\theta_2\cos\eta + B\cos\theta_1\cos\theta_2] + \mathbf{a}_n C[\sin\theta_1\cos\theta_2\cos\eta + \cos\theta_1\sin\theta_2] + \mathbf{a}_p C\sin\theta_1\cos\theta_2\sin\eta\}. \quad (2a)$$

Comparison (1) with Eq. 16, p. 312, gives

$$A = (C+2)/2, \quad B = -\tfrac{1}{2}.$$

Thus the most general equations that satisfy the Ampère postulates I–IV and our postulates (1, p. 306) are as follows:

Force on charged particle

$$\mathbf{F}_2 = -\frac{Q_1Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left\{\frac{\mathbf{a}_r}{2}[(C+2)\sin^2\theta - \cos^2\theta] + \mathbf{a}_y C\sin\theta\cos\theta\right\}, \quad (3)$$

Force on current element

$$d^2\mathbf{F}_2 = \frac{|I_1||I_2|ds_1ds_2}{4\pi\epsilon c^2 r^2}\{\mathbf{a}_r[(C+2)\sin\theta_1\sin\theta_2\cos\eta - \cos\theta_1\cos\theta_2] + \mathbf{a}_n C[\sin\theta_1\cos\theta_2\cos\eta + \cos\theta_1\sin\theta_2] + \mathbf{a}_p C\sin\theta_1\cos\theta_2\sin\eta\}. \quad (4)$$

The constant C appears only in expressions for the Ampère force. The Coulomb, Weber, and Maxwell forces (5) are unaffected by Warburton's criticism.

Equations 3 and 4 must hold for all values of the angles. In particular, let $\eta = \theta_2 = \pi/2$ in Eq. 4. This condition represents a current element $I_2 ds_2$ that is perpendicular to the plane determined by $\mathbf{a}_r$ and $I_1 ds_1$. Then, according to the classical picture, I_1 produces a magnetic flux that at $I_2 ds_2$ is collinear with the element. Thus $\mathbf{v} \times \mathbf{B} = 0$ and there can be no force. But according to Eq. 4,

$$d\mathbf{F}_2 = \mathbf{a}_n \frac{|I_1||I_2|ds_1 ds_2}{4\pi\epsilon c^2 r^2} C \cos\theta_1,$$

which can be zero in general only when $C = 0$.

Introduction of this condition into Eqs. 3 and 4 results in the Gauss equation and the Ampère equation:

$$\left.\begin{aligned} \mathbf{F}_2 &= -\mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left[1 - \frac{3}{2}\cos^2\theta\right], \\ d^2\mathbf{F}_2 &= \mathbf{a}_r \frac{|I_1||I_2|ds_1 ds_2}{4\pi\epsilon c^2 r^2}\left[2\sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2\right] \end{aligned}\right\}. \quad (5)$$

The proof is not very satisfactory, however, since we have treated fictitious current elements as though they had physical significance. It is desirable, therefore, to look for something susceptible to experimental verification.

3. COMPARISON WITH WARBURTON

Equation 3 expresses the force between moving charges in terms of a component in the r-direction (radial) and a component perpendicular to the r-direction. The question then arises as to whether some value of C may give a force in the direction of the velocity.

If $\mathbf{F}$ is in the direction of the velocity $\mathbf{v}$,

$$\frac{F_r}{F_y} = \cot\theta$$

or

$$\frac{(C+2)\sin^2\theta - \cos^2\theta}{C\sin\theta\cos\theta} = \cot\theta.$$

Then

$$C = \frac{3\cos^2\theta - 2}{1 - 2\cos^2\theta}, \quad (6)$$

which requires that a constant be equal to a function of θ. Therefore, the force cannot be in the direction of the velocity.

One may, however, write Eq. 3 with components in the r-direction and the v-direction:

$$\mathbf{F} = -\frac{Q_1Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left\{\frac{\mathbf{a}_r}{2}[(C+2) - 3(C+1)\cos^2\theta] + \mathbf{a}_v\, C\cos\theta\right\}. \quad (3a)$$

Warburton's equation (3) for the Ampère force (Coulomb term and acceleration terms omitted) is (5)

$$\mathbf{F} = -\frac{Q_1Q_2}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2\left\{\mathbf{a}_r\left[(1-D) - \frac{3}{2}(1-2D)\cos^2\theta\right] - \mathbf{a}_v\, 2D\cos\theta\right\}. \quad (7)$$

Equation 7 is identical with Eq. 3 with $D = -C/2$, and thus our general equation for the Ampère force agrees with Warburton's equation which was obtained in a completely different way. But this conclusion applies only to the Ampère force and not to the complete formula of Warburton. Our remarks (2) on the untenability of the complete Warburton equation still apply.

4. THE GRASSMANN EQUATION

Classical electromagnetic theory includes the Grassmann equation for the force between current elements:

$$d^2\mathbf{F}_2 = \frac{|I_1||I_2|}{4\pi\epsilon c^2 r^2}[(d\mathbf{s}_1 \times \mathbf{a}_r) \times d\mathbf{s}_2]. \quad (8)$$

In terms of the angles of Fig. 1, this relation becomes

$$d^2\mathbf{F}_2 = \frac{|I_1||I_2|ds_1ds_2}{4\pi\epsilon c^2 r^2}\{\mathbf{a}_r\sin\theta_1\sin\theta_2\cos\eta - \mathbf{a}_n\sin\theta_1\cos\theta_2\cos\eta - \mathbf{a}_p\sin\theta_1\cos\theta_2\sin\eta\}. \quad (9)$$

Equation 9 satisfies our previous (1) Eq. 16 and thus gives the correct results for close circuits. But it does not agree with Eq. 4 and therefore *it cannot be derived from a force-equation for moving charges*, relativistic in the Galilean sense.

5. IONIC CONDUCTION

Previous equations for the force on current elements have been derived for metallic conduction, where the positive charges are held in the crystal lattice and only electrons are free. Consider now the more

general case where both positive and negative charges move, as in molten metals or in electrolytes. There are now four velocities (v_1^+, v_1^-, v_2^+, v_2^-) instead of the previous two; and there are four combinations ($-+$, $++$, $--$, $+-$) instead of three. The expression corresponding to Eq. 2 is

$$d^2\mathbf{F}_2 = \frac{N_1N_2A_1A_2|Q_e|^2ds_1ds_2}{r^2}\{\mathbf{a}_r[f_r(v_4,\theta_4) - f_r(v_5,\theta_5) - f_r(v_6,\theta_6) + f_r(v_7,\theta_7)] + \mathbf{a}_n[f_y(v_4,\theta_4)\cos\eta_4 - f_y(v_5,\theta_5)\cos\eta_5 - f_y(v_6,\theta_6) \times \cos\eta_6 + f_y(v_7,\theta_7)\cos\eta_7 - f_z(v_4,\theta_4)\sin\eta_4 + f_z(v_5,\theta_5)\sin\eta_5 + f_z(v_6,\theta_6)\sin\eta_6 - f_z(v_7,\theta_7)\sin\eta_7] + \mathbf{a}_p[f_y(v_4,\theta_4)\sin\eta_4 - f_y(v_5,\theta_5) \times \sin\eta_5 - \eta f_y(v_6,\theta_6)\sin\eta_6 + f_y(v_7,\theta_7)\sin\eta_7 + f_z(v_4,\theta_4)\cos\eta_4 - f_z(v_5,\theta_5)\cos\eta_5 - f_z(v_6,\theta_6)\cos\eta_6 + f_z(v_7,\theta_7)\cos\eta_7]\}, \quad (10)$$

where $v_i = |\mathbf{v}_2 - \mathbf{v}_1|$, θ_i is an angle between $\mathbf{v}_i$ and $\mathbf{a}_r$, and η_i is an angle between the plane of $(\mathbf{v}_2,\mathbf{a}_r)$ and the plane of $(\mathbf{v}_i,\mathbf{a}_r)$. The subscripts are 4,5,6,7, corresponding to (v_1^-, v_2^+), (v_1^+,v_2^+), (v_1^-,v_2^-), and (v_1^+,v_2^-).

It was hoped that this more general case might suggest a method of evaluating the constant C of Eq. 4. Actually, the allowable terms were found to be exactly as before, the final equation being

$$d^2\mathbf{F}_2 = \frac{N_1N_2A_1A_2|Q_e|^2ds_1ds_2}{4\pi\epsilon c^2r^2}[v_1^+v_2^+ + v_1^+v_2^- + v_1^-v_2^+ + v_1^-v_2^-]\{P\}, \quad (11)$$

where $\{P\}$ is the bracket of Eq. 4, Also.

$$|I_1| = N_1A_1|Q_e|(v_1^+ + v_1^-), \quad |I_2| = N_2A_2|Q_e|(v_2^+ + v_2^-),$$

or

$$|I_1||I_2| = N_1N_2A_1A_2|Q_e|^2[v_1^+v_2^+ + v_1^+v_2^- + v_1^-v_2^+ + v_1^-v_2^-].$$

Thus Eq. 11 for the general case is identical with Eq. 4 for stationary positive charges. We conclude that Postulate C of the previous paper (1) is unnecessary: Eq. 4 *applies to all uncharged conductors, irrespective of whether the steady current consists of moving positive charges, negative charges, or any combination of the two.* The concept of *current* is a useful one when dealing with current elements or complete circuits. That the concept is not universally applicable, however, is shown in Section 8.

6. CURRENT ELEMENT AND MOVING CHARGE

Consider now the force $\mathbf{F}_2$ on a charge Q_2, caused by a current element I_1ds_1 (Fig. 1 with I_2ds_2 replaced by Q_2). The charged particle has a velocity $\mathbf{v}_2$ with respect to the positive charges in the wire. Then, for the interaction of $+Q_2$ and the positive charges in ds_1, Eq. 3 gives

$$d\mathbf{F}_{++} = -\frac{Q_2 N_1 A_1 |Q_e| ds_1}{4\pi\epsilon c^2 r^2}\left\{\frac{\mathbf{a}_r}{2} v_2^2[(C+2) - (C+3)\cos^2\theta_2] + \mathbf{a}_n C v_2^2 \sin\theta_2 \cos\theta_2\right\}.$$

Similarly,

$$d\mathbf{F}_{-+} = +\frac{Q_2 N_1 A_1 |Q_e| ds_1}{4\pi\epsilon c^2 r^2}\left\{\frac{\mathbf{a}_r}{2} v_3^2[(C+2) - (C+3)\cos^2\theta_3] + \mathbf{a}_n C v_3^2 \sin\theta_3 \cos\theta_3 \cos\eta_3 + \mathbf{a}_p C v_3^2 \sin\theta_3 \cos\theta_3 \sin\eta_3\right\},$$

where

$$v_3^2 = v_1^2 + v_2^2 + 2v_1 v_2 (\sin\theta_1 \sin\theta_2 \cos\eta + \cos\theta_1 \cos\theta_2),$$

$$\cos\theta_3 = \frac{1}{v_3}(v_1 \cos\theta_1 + v_2 \cos\theta_2),$$

$$\sin\theta_3 \cos\eta_3 = \frac{1}{v_3}(v_1 \sin\theta_1 \cos\eta + v_2 \sin\theta_2),$$

$$\sin\theta_3 \sin\eta_3 = \frac{v_1}{v_3}\sin\theta_1 \sin\eta.$$

The total force on Q_2 is the sum of $\mathbf{F}_{++}$ and $\mathbf{F}_{-+}$, or

$$d\mathbf{F}_2 = \frac{Q_2 |I_1| ds_1}{4\pi\epsilon c^2 r^2}\left\{\frac{\mathbf{a}_r}{2}[(C+2)(v_1 \sin^2\theta_1 + 2v_2 \sin\theta_1 \sin\theta_2 \cos\eta) - (v_1 \cos^2\theta_1 + 2v_2 \cos\theta_1 \cos\theta_2)] + \mathbf{a}_n C[v_1 \sin\theta_1 \cos\theta_1 \cos\eta + v_2(\sin\theta_1 \cos\theta_2 \cos\eta + \cos\theta_1 \sin\theta_2) + \mathbf{a}_p C \sin\theta_1 \sin\eta \times (v_1 \cos\theta_1 + v_2 \cos\theta_2)\right\}. \quad (12)$$

For the special case of a stationary charge ($v_2 = 0$), and with $\eta = 0$ for simplicity,

$$d\mathbf{F}_2 = \frac{Q_2 |I_1|^2 ds_1}{4\pi\epsilon c^2 r^2 N_1 A_1 |Q_e|}\left\{\frac{\mathbf{a}_r}{2}[(C+2)\sin^2\theta_1 - \cos^2\theta_1] + \mathbf{a}_n C \sin\theta_1 \cos\theta_1\right\}. \quad (12a)$$

The pecularity of these equations is their prediction of a force when $v_2 = 0$, a prediction that is at variance with classical electromagnetism. Also, the force is not directly proportional to the current in Eq. 12*a* but varies as the square of the current. Evidently, no choice of C will eliminate this force.

If Q_2 is in motion, its velocity $\mathbf{v}_2$ is usually much greater than the low drift velocity $\mathbf{v}_1$ of the electrons. In an electric generator, for instance, v_2 may be 10 m sec^{-1} or more while v_1 rarely exceeds 10^{-3} m sec^{-1}, giving $v_2/v_1 \geqq 10^4$. For $v_2 \gg v_1$ and $\eta = 0$, Eq. 12 reduces to

$$d\mathbf{F}_2 \leqq \frac{Q_2 v_2 |I_1| ds_1}{4\pi\epsilon c^2 r^2} \{\mathbf{a}_r[(C+2)\sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2] + \mathbf{a}_n C \sin(\theta_1 + \theta_2)\}. \quad (12b)$$

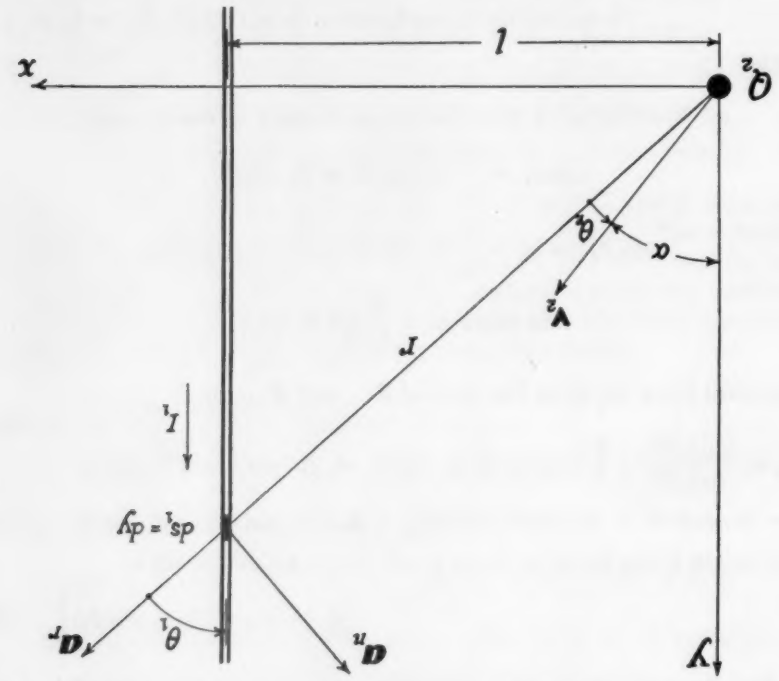

FIG. 2. The long wire in the y-direction carries a current I_1 and exerts a force F_2 on the charge Q_2. The relative velocity of Q_2 with respect to the wire is v_2.

The force on a charged particle, caused by current in any form of circuit, is obtained by integrating the foregoing equations. For example, take a long, straight conductor (Fig. 2) carrying a direct current I_1. Integration of Eq. 12 from $-\infty$ to $+\infty$ gives

$$\mathbf{F}_2 = \frac{Q_2|I_1|}{4\pi\epsilon c^2}\{\mathbf{a}_x(v_1 + 2v_2\cos\alpha) - \mathbf{a}_y 2v_2 \sin\alpha\}. \quad (13)$$

For two parallel wires, carrying I_1 in opposite directions and with Q_2 midway between them,

$$\mathbf{F}_2 = \frac{4Q_2 v_2 |I_1|}{4\pi\epsilon c^2 l}\{\mathbf{a}_x \cos\alpha - \mathbf{a}_y \sin\alpha\}. \quad (14)$$

Note that C has disappeared in both Eqs. 13 and 14; so tests made with this complete circuit cannot possibly determine the constant.

According to classical theory, the force on the charged particle is

$$\mathbf{F}_2 = Q_2\mathbf{v}_2 \times \mathbf{B}. \tag{15}$$

But for a long conductor,

$$\mathbf{B} = \mathbf{a}_z \frac{|I_1|}{2\pi\epsilon c^2 l},$$

$$\mathbf{v}_2 = v_2(\mathbf{a}_x \sin\alpha + \mathbf{a}_y \cos\alpha),$$

where $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are unit vectors. Then Eq. 15 gives

$$\mathbf{F}_2 = \frac{Q_2 v_2 |I_1|}{2\pi\epsilon c^2 l} (\mathbf{a}_x \sin\alpha + \mathbf{a}_y \cos\alpha) \times \mathbf{a}_z$$

$$= \frac{Q_2 v_2 |I_1|}{2\pi\epsilon c^2 l} (\mathbf{a}_x \cos\alpha - \mathbf{a}_y \sin\alpha),$$

which is similar to Eq. 13 but neglects $\mathbf{v}_1$. For two conductors, $\mathbf{B}$ is doubled and Eq. 14 is obtained.

The fact that C disappears in the foregoing special case does not prove that this constant cannot be determined by test with a less-symmetric circuit. Consider the more general case (Fig. 3) of a small

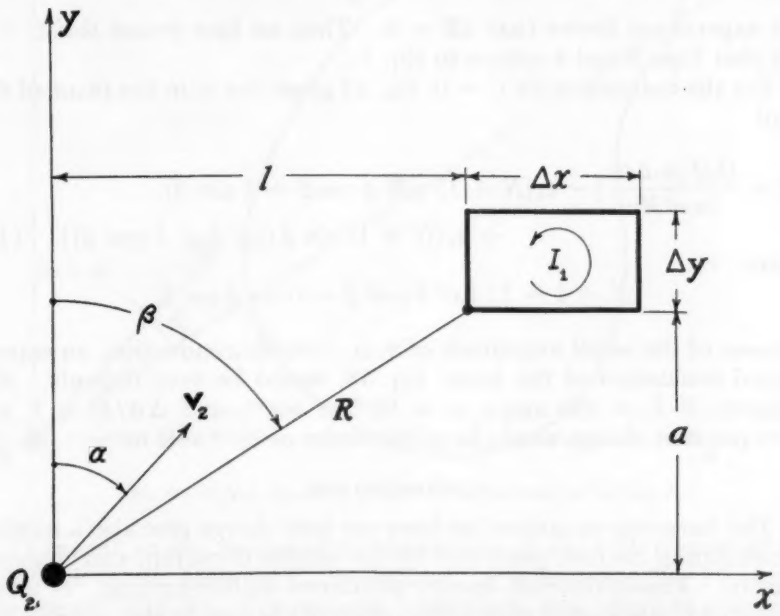

FIG. 3. A current I_1 in the rectangular loop exerts a force F_2 on the charge Q_2.

rectangular loop carrying a current I_1. The case is of fundamental importance, since the effect of a loop of any shape can be obtained by summing the effects of the infinitesimal loops of which it is composed. Direct application of Eq. 12 yields terms in v_2:

$$\Delta\mathbf{F} = \frac{Q_2|I_1|v_2\Delta x\Delta y}{4\pi\epsilon c^2 R^3}[-\mathbf{a}_x \cos\alpha + \mathbf{a}_y \sin\alpha] = \frac{Q_2|I_1|}{4\pi\epsilon c^2 R^3} \Delta\mathbf{A} \times \mathbf{v}_2, \tag{16}$$

which agrees with classical theory. Note that C has again disappeared. Thus even with the most general plane circuit, the force on a charged particle cannot yield any information on the value of C, using terms in $\mathbf{v}_2$.

Equation 16 was obtained for $\mathbf{v}_2$ in the plane of the loop. Now consider that the velocity $\mathbf{v}_2$ is perpendicular to the plane of the loop: $\eta = \theta_2 = \pi/2$. In this case, neglecting v_1, Eq. 12 reduces to the simple form

$$d\mathbf{F} = \mathbf{a}_n \frac{Q_2|I_1|Cv_2}{4\pi\epsilon c^2} \frac{ds_1}{r^2} \cos\theta_1.$$

Integration around the rectangular circuit gives

$$\Delta\mathbf{F} = \mathbf{a}_r \frac{Q_2|I_1|v_2\Delta\mathfrak{a}}{4\pi\epsilon c^2 R^3} C. \tag{17}$$

But experiment shows that $\Delta\mathbf{F} = 0$. Thus *we have proved that* $C = 0$ and that Eqs. 3 and 4 reduce to Eq. 5.

For the v_1-terms, with $C = 0$, Eq. 12 gives (for v_1 in the plane of the loop)

$$\Delta\mathbf{F} = \frac{Q_2|I_1|v_1\Delta\mathfrak{a}}{8\pi\epsilon c^2 R^3} \{-\mathbf{a}_x(N + 15\sin^3\beta\cos\beta + 3\sin^4\beta) + \mathbf{a}_y(N + 15\sin\beta\cos^3\beta + 3\cos^4\beta)\}, \tag{18}$$

where

$$N = 1 - 12\sin^2\beta\cos^2\beta - 6\sin\beta\cos\beta.$$

Because of the small magnitude of $\mathbf{v}_1$ in metallic conduction, an experimental evaluation of the force, Eq. 18, would be very difficult. For instance, if $I_1 = 100$ amps, $v_1 = 10^{-3}$ m sec^{-1}, and $\Delta\mathfrak{a}/R^2 = 1$, the force per unit charge would be of the order of 10^{-8} volt m^{-1}.

7. GENERATED EMF.

The foregoing equations for force per unit charge give also a method of calculating the emf. generated by the motion of current-carrying conductors. This is the emf. usually attributed to flux cutting. A transformer emf., associated with acceleration of charges, is also possible but will not be considered here (6).

In Fig. 4, the current element $I_1 ds_1$ produces a force $d\mathbf{F}_2$ on a charge Q_2, which may be considered to be at the origin of a coordinate system. For $v_2 \gg v_1$ and $\eta = 0$, Eq. 12*b* yields

$$\frac{d\mathbf{F}_2}{Q_2} = \frac{|I_1|v_2 ds_1}{4\pi\epsilon c^2 r^2} \{\mathbf{a}_r[(C + 2)\sin\theta_1\sin\theta_2 - \cos\theta_1\cos\theta_2] + \mathbf{a}_n C\sin(\theta_1 + \theta_2)\}.$$

The generated emf. in an element ds_2, caused by the motion of this element with respect to a current element $I_1 ds_1$, is

$$d^2V_2 = \frac{d\mathbf{F}_2}{Q_2} \cdot d\mathbf{s}_2.$$

Thus the emf. produced in ds_2 is

$$d^2V_2 = \frac{|I_1| v_2 ds_1 ds_2}{4\pi\epsilon c^2 r^2} \{[(C+2)\sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2] \times \cos(\alpha+\theta_2) + C \sin(\theta_1+\theta_2)\sin(\alpha+\theta_2)\}. \quad (19)$$

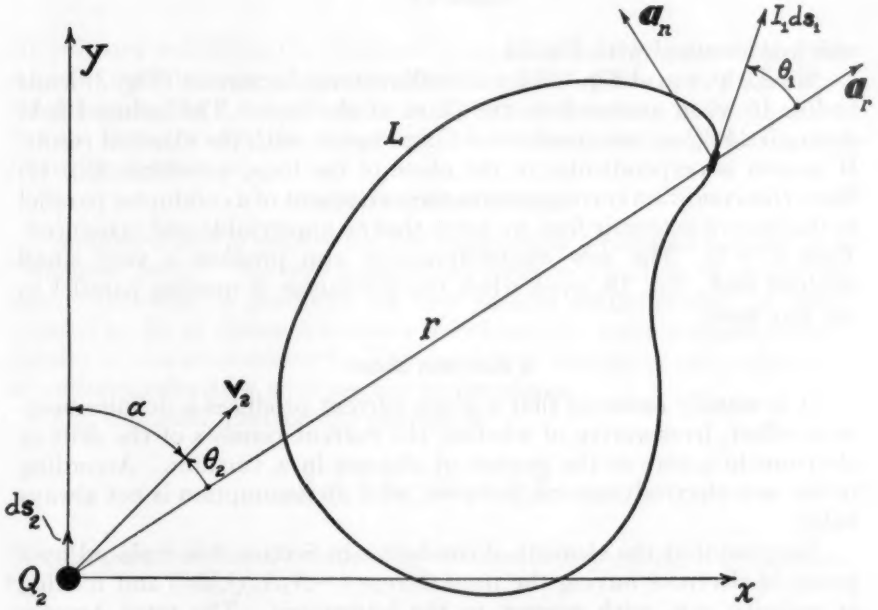

FIG. 4. A test charge Q_2 is acted on by current in the loop L.

The emf. in $d\mathbf{s}_2$, produced by the motion of $d\mathbf{s}_2$ with respect to any complete circuit carrying I_1, is obtained by integrating Eq. 19 around the circuit (Fig. 4). For example, consider a long, straight conductor carrying I_1 in the y-direction. Then $\theta_2 = \theta_1 - \alpha$, and from Eq. 19,

$$dV_2 = \frac{|I_1| v_2 ds_2}{4\pi\epsilon c^2} \int_{-\infty}^{\infty} \{(3C+2)\cos\alpha \sin^2\theta_1 \cos\theta_1 - (2C+3) \times \sin\alpha \sin\theta_1 \cos^2\theta_1 - \cos\alpha \cos^2\theta + C \sin\alpha \sin^2\theta\} \times \frac{dy}{r^2} = \frac{|I_1| v_2 ds_2}{2\pi\epsilon c^2 l} \sin\alpha. \quad (20)$$

Again the result is independent of C.

For a conductor of length b along the y-axis, the generated emf. is therefore

$$V_2 = \frac{|I_1| v_2 b}{2\pi\epsilon c^2 l} \sin\alpha. \quad \text{(volts)} \tag{21}$$

Equation 21 agrees with classical theory. According to Faraday's equation,

$$V_2 = B\, b\, v_2 \sin\alpha$$
$$= \left(\frac{I_1}{2\pi\epsilon c^2 l}\right) b v_2 \sin\alpha,$$

which is identical with Eq. 21.

Similarly, use of Eq. 12b for a small rectangular circuit (Fig. 3) leads to Eq. 16 when motion is in the plane of the loop. The induced field strength $\Delta\mathbf{F}/Q_2$ is independent of C and agrees with the classical result. If motion is perpendicular to the plane of the loop, we obtain Eq. 17. Since this condition corresponds to the movement of a conductor parallel to the lines of magnetic flux, we know that no appreciable emf. is induced. Thus $C = 0$. The new electrodynamics also predicts a very small induced emf., Eq. 18, even when the conductor is moving parallel to the flux lines.

8. ELECTRON BEAMS

It is usually assumed that a given current produces a definite magnetic effect, irrespective of whether the current consists of the drift of electrons in a wire or the motion of charges in a vacuum. According to the new electrodynamics, however, such an assumption is not always valid.

Suppose that the element of conductor in Section 6 is replaced by a group of electrons having the total charge $(-N_1A_1|Q_e|ds_1)$ and moving at velocity $-\mathbf{v}_1$ with respect to the laboratory. The total Ampère force on Q_2 is then merely $d\mathbf{F}_{-+}$ of Section 6, instead of the sum of $d\mathbf{F}_{++}$ and $d\mathbf{F}_{-+}$. Thus

$$d\mathbf{F}_2 = +\frac{Q_2 N_1 A_1 |Q_e| ds_1}{4\pi\epsilon c^2 r^2}\left\{\frac{\mathbf{a}_r}{2}[(C+2)(v_1^2\sin^2\theta_1 + v_2^2\sin^2\theta_2 + 2v_1v_2 \times \sin\theta_1\sin\theta_2\cos\eta) - (v_1\cos\theta_1 + v_2\cos\theta_2)^2] + \mathbf{a}_n C[v_1^2\sin\theta_1 \times \cos\theta_1\cos\eta + v_2^2\sin\theta_2\cos\theta_2 + v_1v_2(\sin\theta_1\cos\theta_2\cos\eta + \cos\theta_1 \times \sin\theta_2)] + \mathbf{a}_p C v_1 \sin\theta_1 \sin\eta(v_1\cos\theta_1 + v_2\cos\theta_2)\right\}. \tag{22}$$

This is certainly not the force that was obtained when the same current was in a wire, Eq. 12. Indeed, the concepts of current and current density appear to be untenable in such examples.

In Eq. 22, v_1 is no longer the small velocity of electron drift but may have very high values. If $v_1 \gg v_2$, $\eta = 0$, and $C = 0$, Eq. 22 becomes

$$\mathbf{F}_2 = -\mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}\left(\frac{v_1}{c}\right)^2\left[1 - \frac{3}{2}\cos^2\theta_1\right], \tag{22a}$$

which is equivalent to Eq. 1. These equations are for the Ampère force. Since the "conductor" is now charged, however, there is also a Coulomb force,

$$\mathbf{F}_2 = -\mathbf{a}_r \frac{Q_1 Q_2}{4\pi\epsilon r^2}. \tag{23}$$

At ordinary velocities, the Coulomb force is immensely greater than the Ampère force because of the $(v_1/c)^2$ in Eq. 22*a*. With $v_1 = 1$ km sec^{-1}, for instance, the Coulomb force is 10^{11} times as great as the Ampère force. But if $v \to c$, the Ampère force is important, as shown in the following section.

9. PARTICLE IN ELECTRIC FIELD

Sections 5 and 6 dealt with the behavior of a charged particle in a "magnetic field." We now consider the force on a particle in an electric field. The field is produced by two parallel metal plates. A large plate (Fig. 5), at distance h above the charge Q_2, has a uniform charge density of $+\omega$ coulomb m^{-2}. The particle is moving in the y-direction at uniform velocity $\mathbf{v}$ with respect to the plates.

The Coulomb force on the particle is

$$\mathbf{F} = -\mathbf{a}_z \frac{\omega}{\epsilon} Q_2 = Q_2\mathbf{E}. \tag{24}$$

The Ampère force on Q_2, caused by an elementary charge d^2Q_1 is, from Eq. 3,

$$d^2\mathbf{F}_2 = -\frac{Q_2\omega}{4\pi\epsilon r^2}\left(\frac{v}{c}\right)^2 \xi\, d\xi\, d\varphi \left\{\frac{\mathbf{a}_r}{2}[(C+2)\sin^2\theta - \cos^2\theta] + \mathbf{a}_n C \sin\theta\cos\theta\right\}.$$

The total effect of the plate is obtained by integrating the above expression, or

$$\mathbf{F}_2 = -\mathbf{a}_z \frac{Q_2\omega}{4\pi\epsilon}\left(\frac{v}{c}\right)^2 \int_{-\pi/2}^{\pi/2}\int_0^\infty \{(C+2)h^3\xi + (C+2)h\xi^3\cos^2\varphi - (2C+1)h\xi^3\sin^2\varphi\}\frac{d\xi\, d\varphi}{r^5} = -\mathbf{a}_z \frac{Q_2\omega}{4\epsilon}\left(\frac{v}{c}\right)^2 = \frac{1}{4}\left(\frac{v}{c}\right)^2\mathbf{E}. \tag{25}$$

The result is independent of C and independent of h.

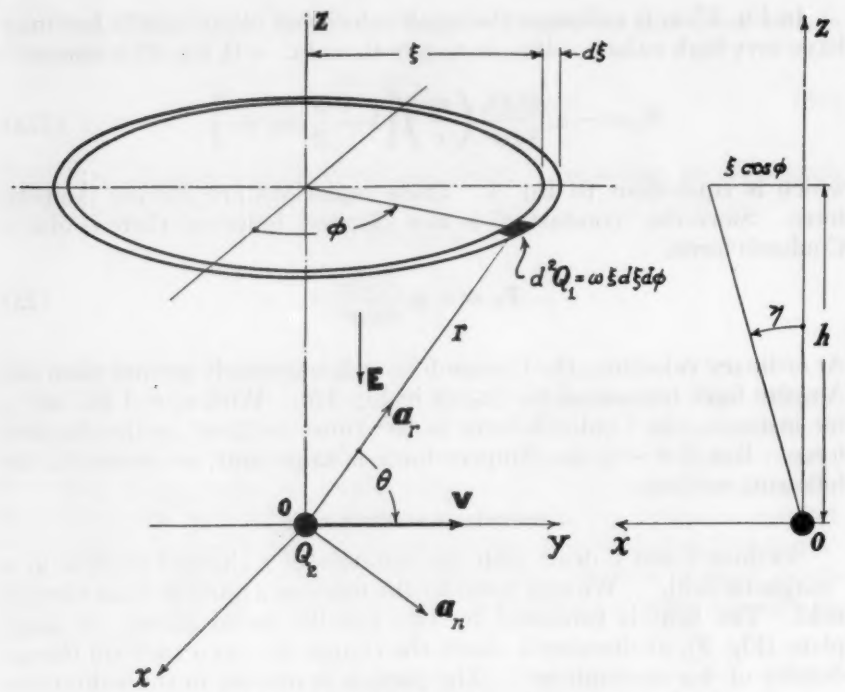

FIG. 5. A charged plate parallel to the xy-plane acts on a charge Q_2 which is moving at constant velocity $\mathbf{v}$.

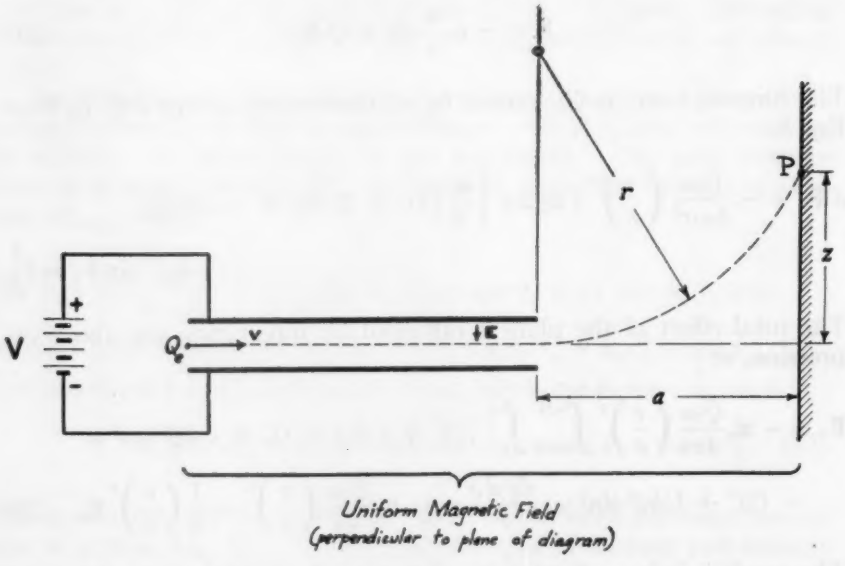

FIG. 6. Schematic diagram for measurement of the Kaufmann effect.

For the two parallel plates of the capacitor, the force is doubled. The total force on Q_2 is the sum of the Coulomb and Ampère forces, or

$$\mathbf{F}_2/Q_2 = \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 \right]\mathbf{E}. \tag{26}$$

Classical theory assumes, quite without justification, that the force on a charged particle in an electric field is always $Q\,\mathbf{E}$, irrespective of the velocity or acceleration of the particle. Equation 26, on the contrary, introduces an additional term which is proportional to the square of the velocity. This correction term is a consequence of the basic electrodynamic relation, Eq. 3, which is based on the Ampère experiments and other low-velocity phenomena. One might expect that for $v \to c$, additional correction terms would be required.

In the measurement of the Kaufmann effect (7) high-velocity electrons pass between the plates of a capacitor, Fig. 6, and impinge on a photographic film at P. The whole space is in a uniform magnetic field which is perpendicular to the plane of the diagram. The crossed fields between the closely spaced capacitor plates act as a velocity-sorter, so $\mathbf{v}$ for the emerging electrons is essentially a constant, both in magnitude and direction.

Let us apply classical theory to Fig. 6. In the crossed fields between plates, the electric and magnetic forces are equal and opposite, so

$$\mathbf{F}_2 = Q_2\mathbf{E} = -\,Q_2\,\mathbf{v}_c \times \mathbf{B}$$

or

$$|v_c| = E/B. \tag{27}$$

In the magnetic field outside the plates, the electron moves in a circular arc of radius r, and

$$F = Q_2 v_c\, B = m\, v_c^2/r$$

or

$$r = \frac{m\, v_c}{Q_2 B}. \tag{28}$$

Experiment does not agree with this prediction of classical theory. To patch up the difficulty, one assumes that mass is a function of velocity:

$$m = m_0[1 - (v/c)^2]^{-\frac{1}{2}},$$

where m is the "longitudinal mass" and m_0 is the "rest mass" of the particle. Then Eq. 28 becomes

$$r = \frac{m_0 E}{Q_2 B^2[1 - (v/c)^2]^{\frac{1}{2}}} = \frac{m_0 E}{Q_2 B^2}\left[1 + \frac{1}{2}(v/c)^2 + \frac{3}{8}(v/c)^4 + \cdots \right]. \tag{28a}$$

According to the new electrodynamics, equality of forces in the crossed fields gives

$$F_z = Q_2 E\left[1 + \frac{1}{2}(v/c)^2\right] = Q_2 vB$$

or

$$v = \frac{E}{B}\left[1 + \frac{1}{2}(v/c)^2\right]. \tag{26a}$$

In the magnetic field, the high-velocity electrons act as in classical theory, so

$$r = \frac{m_0 v}{Q_2 B} = \frac{m_0 E}{Q_2 B^2}\left[1 + \frac{1}{2}(v/c)^2\right], \tag{26b}$$

which agrees with experiment (and Eq. 28*a*) up to fourth-order terms. By adding terms in $(v/c)^{2n}$ to Eq. 3, we could fit all the high-velocity data without affecting any of the ordinary results.

Thus the Kaufmann effect does not require relativistic mass but is a direct consequence of the presence of the Ampère force, which is ignored in classical theory. Mass is an invariant. This fact has been discussed by Bush (**8**) and by O'Rahilly (**9**).

10. CONCLUSIONS

Two questions have been considered: the value of C in Eqs. 3 and 4, and the relation between the predictions of classical electromagnetic theory and the new electrodynamics. Regarding the first question, we have proved that $C = 0$. This reduces Eqs. 3 and 4 to the comparatively simple expressions of Gauss and Ampère, Eq. 5. All of our previous results (**5**) remain as originally stated.

The second question is answered by comparing the generally accepted findings of electromagnetic theory with the predictions of the new electrodynamics. In most cases, the results are identical. Several differences occur, however, among which may be mentioned

(1) The Grassmann equation (Section 4) is not in accord with electron theory.
(2) Classical theory states that a constant magnetic field exerts no force on a stationary electric charge. The new electrodynamics predicts such a force, though this force is probably too small to detect (Section 6).
(3) Classical theory states that no emf. is induced in a conductor that moves along a flux line. The new electrodynamics predicts an emf. which is probably too small to be measured (Section 7).
(4) It is customary to assume that the force on a moving charge in a magnetic field is the same, whether this field is produced by current

in a wire or by an electron beam. According to the new electrodynamics, this assumption is false (Section 8).

(5) The predictions of classical theory fail for high-velocity particles, and the theory must be modified by the assumption that mass is a function of velocity. This difficulty is a result of the neglection of the Ampère force by Maxwell's theory. According to the new electrodynamics, *force* is a function of velocity but *mass* is invariant (Section 9).

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Bug Juice (*Industrial Bulletin* of Arthur D. Little, Inc., No. 322).—Each year some 2.5 trillion insects are born, live, and die for no apparent purpose other than man's pleasure and comfort. With a minimum of human assistance, these bugs produce over 50 million pounds of a resinous material that man has never been able to match. Man has, however, used this "bug juice" in a refined form known as shellac, for perhaps 4000 years.

During World War II when supplies of shellac were cut off, one company developed a protein-based substitute that satisfied many users. This product, Zinlac, and a few others introduced later, but in lesser quantities, kept the shellac industry going for about five years. When the natural product returned to world markets, substitutes were dropped, but the technology remains available.

Crude lac is a scale-like, amber colored, waxy, resinous substance exuded by *Tachardia lacca* to serve as a protective coating for the insect's soft body during the major part of its six-month life span. The insects live as parasites on trees, chiefly in India, Thailand, and Burma. The lac is "harvested" four times a year from the branches to which it adheres. Once separated from the twigs, the crude lac is ground, washed, dried and graded according to particle size. The coarsest size, known as seedlac, is purified and refined by heat or solvents into various grades of commercial shellac. About half the world production is consumed in the United States.

The most familiar uses for shellac are as a quick-drying, hard-wearing finish for wood, and as a sealer coat prior to finishing. Shellac is soluble in quick-drying solvents like the lower alcohols, and from these solutions, thin, tough coatings can be formed in and on the surface of the wood. Shellac is also used to give better adhesion and toughness to nitrocellulose lacquers for such difficult surfaces as some metals and plastics.

Perhaps the largest single use for bleached dewaxed shellac is in self-polishing floor waxes; a typical formulation is an aqueous solution of bleached, dewaxed shellac into which a small amount of modifying wax has been emulsified. This type of "high resin" formulation came along at about the same time that vinyl plastic took over the field of phonograph records, an end use that had threatened to monopolize the entire world supply of shellac.

Shellac can be found in a number of lower volume, but nevertheless important, uses. It is used in the preparation of acid resists for zinc printing plates and in inks for printing on such smooth, non-porous surfaces as glassine, metal foil, and so forth. It is also used for cementing felt and wool in hat manufacture, for leather finishing, for protecting the silver backing on mirrors, for setting coiffures, and for bonding the abrasive particles in grinding wheels. Its excellent electrical properties make it especially suitable as a molding material for insulating parts, including bonded mica sheets. Some highly refined shellac is used to impart a colorless, tasteless glaze to candies to prevent sticking, but its use in pharmaceuticals is of greater importance. Pills have long been coated with shellac for improved appearance or to hide unpleasant taste. More recently, shellac has been used as a coating for pills that must pass from the stomach to the intestines without dissolving, since shellac has excellent resistance to acids found in the stomach.

NUCLEAR ENERGY NEWS

AEC ACCESS PERMITS

The Access Permit system (see this JOURNAL, Vol. 260, p. 129) put into effect by the Atomic Energy Commission is being made use of by a large variety of industrial and research organizations. Periodically the Commission has announced lists of those to whom access permits have been granted. The Franklin Institute is included in the third such group which was announced on August 16, by which time a total of 229 permits had been issued.

FOOD PRESERVATION WITH GAMMA RAYS

Argonne National Laboratory placed in operation on August 6, a large irradiation facility for studies of the effects of gamma radiation on foods and other materials. Simultaneously, the Laboratory announced the availability of this new facility to non-Laboratory users as a part of its program of providing assistance to industrial organizations, educational institutions, and Government agencies for the development of peacetime uses of atomic energy. Argonne National Laboratory is operated for the U. S. Atomic Energy Commission by The University of Chicago.

One of the first users will be the Army Quartermaster Corps, which has recently begun a broad study of the sterilization of food by irradiation. The long-range objective is to determine if foods can be preserved by irradiation as a substitute for present treatment by heat, chemicals, and other processing without detracting from their natural color, flavor, odor, and texture. Although the first major use of the facility will be concerned with the study of food preservation, the new facility is not limited to such studies. Other materials which do not exceed 4 in. in diameter and 28 in. in length can be accommodated.

The irradiation facility makes use of highly radioactive uranium fuel elements which have been discharged from the AEC's Materials Testing Reactor. Each fuel element provides about 70,000 curies of gamma radiation. Experiments are conducted in a metal rack which holds up to twelve fuel elements in a vertical position. Between the fuel elements are spaces for the materials to be irradiated. The rack rests on the floor of a 55,000 gal. pool of water, which is about 20 ft. deep, 30 ft. long and 15 ft. wide. The water serves to shield the personnel from the radiation being emitted by the fuel elements.

The operational procedure for food irradiations is as follows: the material is sealed in No. 2 cans prior to delivery to the irradiation

facility. Here, the cans are placed in twelve aluminum-stainless steel cylinders or urns which can accommodate five cans each. The loaded cylinders are then lowered into the water between the fuel elements, where they are rotated slowly during the irradiation period, to insure uniform exposure to the gamma rays.

LICENSES FOR REACTORS

The first license to be issued by the AEC for the operation of a research reactor was issued to the Pennsylvania State University on July 8. This license for the University's "swimming pool" type reactor is for a ten-year period. It authorizes the University to receive and use enriched uranium as fuel and also to possess such radioactive materials as result from operation of the reactor. Operation of the reactor at power levels not in excess of 30 watts is authorized during an initial trial period. After the characteristics of the reactor have been determined and the suitability of control and safety features have been established, it is contemplated that operation at a higher power level will be authorized. The AEC has approved proposed rules of practice applying to its licensing program. The rules were designed to safeguard the interests of licensees and applicants for licenses and were prepared to comply with the Administrative Procedure Act. The proposed rules will govern the conduct of proceedings before the Commission involving licensing and licenses, and such matters as amendments, revocations, suspensions, transfers and renewals of licenses; intervention in such proceedings by interested persons; and how and when hearings may be obtained.

MEDICAL RESEARCH REACTOR

The U. S. Atomic Energy Commission has entered into a research contract with the University of California under which the university will construct a nuclear reactor specifically designed for medical treatment and research. Nuclear reactors have proved to be important tools for medical research and therapy. None of the reactors in existence at present, however, was built specifically for use in medicine.

The Commission will contribute $75,000 towards the project and will support an extensive program of research utilizing the reactor. The Commission also will make available enriched uranium as fuel for the reactor, which will be constructed at the new medical center on the campus of the University of California at Los Angeles. The University will construct a building to house the reactor, at an estimated cost of $400,000.

The reactor will be designed and built by the North American Aviation Corporation for the university. It will be a low-power water boiler reactor, using a solution of uranyl sulfate as fuel. It will operate at a power level of about 5 kilowatts of heat, with a maximum power of 50 kilowatts, and will provide a neutron flux of up to one billion

neutrons per square centimeter per second. It will also furnish a beam of gamma rays. Both can be used in treating deep-seated cancers, often difficult to treat by surgery and present forms of radiation.

The reactor will be dedicated to the treatment of human patients and the training of students in radiation therapy and reactor techniques and theory relative to the field of medicine. It will give the West Coast its first source of slow and fast neutrons sufficient for experimental work with animals and for treatment of human beings. It also will be a source of short-lived isotopes for experimental biology and medicine.

REACTORS FOR SUBMARINES

The submarine *Sea Wolf*, which is the second nuclear powered submarine, was launched at Groton, Conn. on July 21. The power plants for the *Sea Wolf* and *Nautilus* were started at about the same time as parallel and competitive efforts, using reactor designs of marked difference. The reactor for the *Nautilis*, developed by Westinghouse, uses low velocity neutrons, and the heat is removed by water under high pressure. On the other hand, the reactor for the *Sea Wolf*, developed by General Electric, operates with neutrons of higher velocities and liquid metal is the medium of heat transfer.

A month later the Atomic Energy Commission announced that a contract had been awarded to Combustion Engineering Corporation of New York City for the design and development of a reactor suitable for installation in a smaller submarine. This contract calls for the design, manufacture, assembly and test of a reactor and related equipment for this use. The contractor will bear 25 per cent of the cost of the research and development, up to a maximum of $3,000,000.

ATOMIC-ELECTRIC POWER

The first community to receive its entire supply of electric power from a nuclear source was Arco, Idaho on July 17. Electricity produced in an experimental nuclear power plant (the "Boiling Reactor Experiment") at the AEC's National Reactor Testing Station 20 miles from Arco, was fed into transmission lines supplying the electrical appliances of its 1200 inhabitants. This demonstration experiment lasted for more than an hour. A report of it supplemented a description of the reactor that was presented to the International Conference on Peaceful Use of Atomic Energy at Geneva the following month.

The reactor consists of a pressure vessel containing an assemblage of enriched uranium-bearing plates submerged in water, plus a number neutron-absorbing control rods. The water circulates through the reactor core by natural convection. Steam, produced by the heat created by the fissioning of uranium atoms, is conducted to the 3500-kilowatt turbo-generator, located in a nearby building. The simplicity of construction, ease of operation, low cost, and high degree of safety

suggest the possibility that this type of small power plant may be suitable for use in remote areas or in conjunction with mining or manufacturing operations.

The day after the Arco experiment the first atomic electric energy to be sold by the Government was introduced into a public utility system. Steam from the prototype reactor for the submarine *Sea Wolf* was used to turn a 10,000-kilowatt turbo-generator. The electric energy was sold by the AEC to the Niagara-Mohawk Power Corporation for use in the neighborhood of West Milton, New York, where the reactor is under test. A price of 3 mills per kilowatt hour was established by the AEC; but it was emphasized that this operation was an experiment and that this price was not necessarily a measure of the true cost of the electricity.

Previously classified details of the reactor to be used at Shippingport, Pennsylvania, for the first full-scale nuclear power plant were revealed at the Geneva Conference. This plant, being constructed by the Westinghouse Electric Corporation and to be operated by the Duquesne Light Company, will contain a 100,000-kilowatt turbo-generator. It is scheduled to deliver at least 60,000 kilowatts of electricity to the Pittsburgh area in 1957. The reactor will be a pressurized water type, operating at a pressure of 2000 psi. The enriched uranium fuel will be made in the form of plates clad with a zirconium alloy immersed in the pressure vessel. There will also be 12 tons of natural uranium in the form of a "blanket" in which plutonium will be created by reaction of neutrons with uranium 238. Subsequently fission of the plutonium will provide a substantial fraction of the power produced.

DECLASSIFICATION OF URANIUM ORE PROCESSING

The Atomic Energy Commission has declassified all technical information on currently-used extraction processes for recovering uranium from uranium-bearing materials to produce unrefined uranium concentrates.

The Commission determined that the information no longer has security sensitivity, and that its declassification would benefit the Commission's raw materials program by encouraging private enterprise to participate in the program and to develop cheaper and more efficient methods of processing uranium ores.

The declassification action does not include information relating to the refinement of uranium concentrates into highly purified forms, nor does it include technical information on any new and important extraction processes which may be developed.

Unrefined uranium concentrates are produced in twelve processing plants in Colorado, Utah, New Mexico, Florida and Illinois. Four additional plants are under construction. All but one of the plants are privately owned and operated.

REGULATIONS FOR PROTECTION AGAINST RADIATION

The Atomic Energy Commission has issued a proposed regulation establishing standards for protection of personnel and the public against radiation hazards. The regulation will apply to all persons who receive, possess, use, or transfer source material, special nuclear material, or by-product material under a general or specific license.

The regulation establishes maximum permissible limits on exposures to external radiation and concentrations of radioactive material, and maximum permissible levels of radiation in areas access to which is not controlled by the licensee. It also establishes controls over the release of radioactive material from licensed facilities. Maximum permissible average concentrations of 71 radioactive isotopes in both air and water are listed in an appendix.

The standards are based on recommendations made by the National Committee on Radiation Protection. On the basis of present scientific knowledge and experience the permissible levels of exposure and concentration and other requirements will provide a substantial margin of safety for personnel and the public.

Other provisions of the regulation include requirements regarding personnel monitoring, protective masks and clothing, caution signs and signals, waste disposal, storage of licensed material, instruction of personnel on safe procedures for handling and using licensed material, and records and reports.

SCINTILLATION COUNTER FOR HUMAN SUBJECTS

The Los Alamos Scientific Laboratory has built a scintillation detector large enough to accommodate a human body and to measure its accumulated amount of radioactivity. By use of this scintillation detector or "human counter" it is possible to measure radioactivity naturally present in the body. All persons have in their bodies naturally occurring radioactivities, principally potassium and carbon. Study of this natural level of radioactivity is valuable in determining how much radiation exposure is permissible without damage.

It is also necessary to guard against the possible ingestion and inhalation of radioactivity for the protection of personnel working with radioactive materials. One of the important steps is the determination of the amount of radioactivity which might have accumulated in the bodies of exposed personnel. The simplest method of determining this, in the case of radioactive materials which emit penetrating gamma rays, is direct measurement of gamma rays from the human body. It is necessary that the detector be large enough to completely enclose the body, in order to obtain the necessary sensitivity to permit measuring the body burden of potentially dangerous gamma-emitting materials while their concentration is still below permissible levels.

The scintillation detector is a large cylindrical tank, six feet long and 28 inches in diameter. Through it runs a cavity large enough to

contain the human subject to be measured. The walls of the tank contain a liquid which gives off minute flashes of light when gamma rays from the person pass through it. This light is detected and amplified by 108 photomultiplier tubes installed in the outer wall of the tank. Electronic instruments record the number of gamma rays registered. The entire tank is surrounded by a ten-ton lead shield to reduce interference by external radioactivities.

GIFTS OF TECHNICAL LIBRARIES

Technical libraries of non-classified data on nuclear energy and its applications have been shipped by the Atomic Energy Commission to 26 nations. These comprehensive collections, each containing the equivalent of documents which would fill 250 feet of library shelving, were developed by the AEC Technical Information Service as one of the several Commission projects supporting the President's Atoms-for-Peace program.

Each gift library, weighing approximately 1,000 pounds, consists of approximately 6500 AEC research and Development Reports, 5000 of which are on microcards; 22 miscellaneous books; 34 bound volumes of scientific and technical texts on nuclear theory; and 11 bound volumes of abstracts of some 50,000 reports and articles published in this country and abroad. Each collection also will include about 55,000 index cards.

The individual collections duplicate material now available in 42 repository libraries in the United States, three in the United Kingdom and one each in Belgium and Canada. The countries to which the libraries were shipped are: Argentina, Australia, Austria, Burma, China (Republic of), Denmark, Egypt, France, Finland, Greece, India, Israel, Italy, Japan, Lebanon, Netherlands, New Zeland, Norway, Peru, Philippines, Portugal, South Africa, Spain, Sweden, Switzerland, and Turkey.

INSURANCE AGAINST NUCLEAR DISASTER

The Insurance Study Group, consisting of ten representatives of the fire and casualty insurance industry, was appointed by the Atomic Energy Commission in March 1955 to review insurance problems created by expanded industrial participation in atomic energy. The interim conclusions presented in a report of its preliminary survey are optimistic. They indicate that after further evaluation of the special factors involved, the insurance industry will be able to provide adequate amounts of most of the insurance required for industrial nuclear activities.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS

CONTINUOUS RECORDING OF SOUND TRANSMISSION LOSS

The acoustical properties of construction materials play an important part in the design of homes, offices, and factories. For effective sound-proofing design at lowest possible cost, architects and engineers require comprehensive data in this field. One of the most critical of acoustical properties is the sound transmission loss (STL)—a factor directly related to the sound attenuating qualities of a material.

Recently the National Bureau of Standards developed a rapid, accurate method for determining the STL of walls over a wide range of frequencies. The method utilizes a commercial recorder which has been adapted to present a continuous record of the difference in decibels between the sound levels in two rooms separated by a test wall. The adaptation was made by R. V. Waterhouse and R. K. Cook of the Bureau's sound laboratory.

In measuring the STL of walls, floors, and other structures, it is often desirable to obtain an accurate curve of STL versus frequency over the range of 100–4000 cps. Usually STL figures are measured with a number of discrete noise bands, each centered at a particular frequency. An approximation of the STL curve of the test panel can be obtained by plotting these data and connecting the points with straight lines. Such an approximation is satisfactory for most purposes, but in studying certain aspects of the transmission of sound through panels—for example, edge or coincidence effects—a more accurate procedure is needed.

A partial solution to this problem is obtained as follows: The panel to be tested is used as a wall common to two adjacent rooms. The room containing the sound-producing equipment is called the *loud room*; the other is called the *quiet room*. Sound apparatus records the average sound pressure level in each test room as a continuous function of frequency, and the STL of the test panel is determined by subtracting the quiet room curve from the loud room curve. However, it is much more convenient to be able to record directly in decibels the difference between the sound levels in the two test rooms. This can be done by the Bureau's method, which presents the STL of the test panel as a chart reading.

The self-balancing commercial recorder used in the NBS apparatus is ordinarily wired as a voltmeter. In this arrangement the voltage to be measured is balanced against a known voltage from a standard cell contained within the instrument. The unknown voltage is fed across

a resistance potentiometer, and part of it is tapped off and compared with the standard voltage. The difference voltage is applied to an amplifier whose output actuates a motor. The motor drives the sliding contact of the potentiometer in the direction that minimizes the difference voltage until at the balance point the two voltages are equal. The position of the sliding contact is recorded as the value of the unknown voltage.

Two modifications were performed to make the instrument record the difference in decibels between two signals. First, the standard cell was removed, and one of the incoming signals used in its place. Second, a logarithmic attenuator was installed in place of the linear slide wire. The attenuator has 40 steps of one db each, set around in a circle. Thus the recorded plot covers the range 0 to 40 db linearly. The scale can be easily changed from this range to other ranges by inserting a fixed attenuator, or by adjusting the gain of the amplifiers.

In operation, the loud and quiet rooms are separated by the panel whose sound transmission loss is to be measured. An oscillator in the loud room supplies a warbled signal whose center frequency is motor-driven over the required frequency range. Sounds from the two rooms are picked up by microphones and fed into separate channels which amplify, filter, and rectify the associated microphone voltages. The logarithmic attenuator continuously reduces the voltage from the loud room to the value of the voltage from the quiet room. The two voltages are automatically balanced at an attenuator setting which is determined by the STL of the panel.

To record accurate differences in decibels, the gain of the amplifiers of both channels (quiet and loud) must not vary differentially during the measurement. In the setup used at NBS a stabilized power supply makes errors from this source negligible.

Data taken with this equipment differ slightly from the true STL curves. The difference lies in the quiet-room absorption correction and the variation in response of the microphones in the two rooms. By suitable choice of microphones, the two corrections can be made to cancel out partially, so that the final correction need not be more than 2 db. This is small enough to ignore in assessing the main features of the STL curve and, if necessary, could be corrected for by an electrical network inserted in one channel.

AN AUTOMATIC UNDERWATER VELOCIMETER

The National Bureau of Standards has developed an instrument that automatically measures the speed of sound in the sea to depths as great as 300 ft. and plots the result as a function of depth or time. The instrument was designed and constructed in the Bureau's sound laboratory by M. Greenspan and C. E. Tschiegg under the sponsorship of the Office of Naval Research. Because of its high accuracy and

almost instantaneous response, the velocimeter is expected to be a useful addition to underwater signalling and detecting apparatus. It should also prove to be a valuable research instrument in oceanography.

The speed of sound in large natural bodies of water varies from about 4600 to 5140 ft. per sec. These variations occur with changes in temperature and, to a lesser extent, with changes in the salinity of the water. Sound velocity also increases about 1 foot per second for each 55-ft. increase in depth. Several other factors, not all of which are well understood, influence the velocity of sound in the sea.

The sound velocity gradients due to these variables produce curvatures in the rays of sound being propagated in the sea. In some instances, reflections are produced if, as is often the case, the thermal or salinity gradients are abrupt. In current practice an estimate of the sound velocity is calculated from the measured temperature and an assumed salinity. The NBS velocimeter, on the other hand, gives an almost instantaneous meter reading of the actual sound velocity.

The NBS velocimeter consists essentially of a pair of piezoelectric transducers of polarized barium-calcium-lead titanate and a reflector, mounted to form a sound path of fixed length. The sending transducer is connected to a pulse generator, and the receiving transducer provides the input for a high-gain pulse-shaping amplifier. The amplifier output retriggers the pulse generator, which then applies another pulse to the sender. The sender in turn produces in the water a sound pulse to actuate the receiver. Thus the sytem continually regenerates a sound pulse whose repetition rate, or frequency, depends on the time it takes the pulse to move through the water. Since the path length is fixed, the frequency depends only on the speed of sound through the water and on the circuit delays. Any variations in sound velocity are recorded as variations in the operating frequency of the velocimeter.

The electrical pulse applied to the sender has a very short rise time, and its leading edge is a precise reference point for the location of the pulse in time. On the other hand, the received pulse is much distorted and rises relatively slowly because of selective attenuation by the water. The high-frequency components of the pulse, which control the rise rate, are attenuated to a greater extent than are the low-frequency components. The fast rise is restored by amplification; nevertheless, there is introduced an unknown delay equal to the time which the received pulse spends below the noise level. This delay, which adds to the various circuit delays, depends on the attenuation of the liquid. For this reason, the velocimeter must be calibrated and used on the same or similar liquids. Since the attenuation characteristics of sea water are almost independent of salinity, the instrument may be calibrated in pure water for which the speed of sound is accurately known as a function of temperature.

The maximum rate of pulse rise at the receiver is obtained with a

straight path terminated by parallel transducers. However, under these conditions a system of multiple echoes exists in the water unless the path is inconveniently long. All echoes arising from a single primary pulse are self-synchronous; however, because of the circuit delays, they are not synchronous with those arising from any other primary pulse. These multiple echoes interfere with the proper operation of the instrument.

To eliminate this effect, the pulse path is folded back on itself by means of a plane reflector which absorb part of the energy of the pulse. Hard rubber and certain plastics are suitable materials for the reflector in the case of a 10-cm. path. The use of the two-way path minimizes errors which arise from the mass motion of the water and also makes possible a more compact device.

In recent performance tests, the NBS velocimeter has been used successfully to plot water temperature and salinity against depth in Chesapeake Bay. The velocimeter can also be used as an underwater control device in other ocean studies.

THE FRANKLIN INSTITUTE

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 403 items have been added this month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays, and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

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MARIN, JOSEPH AND A. B. WISEMAN: Triaxial Plastic Stress-Strain Relations for an Aluminum Alloy 14ST–4.
HARGENS, C. W.: Wireless Sound Systems.
MOON, PARRY AND DOMINA EBERLE SPENCER: Some Electromagnetic Paradoxes.
DANFORTH, W. E. AND J. H. BODINE: Polarization in Thorium Oxide Crystals.
PENDER, HAROLD: Laplacian Transforms and Unit Singularity Functions.
GOLDWATER, J. H.: Effects of Electrolysis in the Thorium Oxide Emitter.

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

METAL COINAGE

Oscar Wilde once wrote: "When I was young I used to think that money was the most important thing in life. Now that I am old, I know it is." If one is to judge from the expressions on the faces of young and old who operate the old coining press in the Museum, a great many other people are of the same opinion as Wilde. Not that in operating the machine they get any money out of it, but they can can get a souvenir coin that looks like a cent. All they have to do is drop a nickel into the slot, turn a lever, and watch for results. There is a grinding of wheels, followed by the low thump of the press, and out comes the newly minted coin, bearing the head of Benjamin Franklin.

Metal coinage dates back to Biblical times but when the Colonies were first established there were no means of producing coins. In order to obtain the goods they required the colonists bartered wampum, powder, beads, and bullets. This was inconvenient, but not as cumbersome as the cattle that is the capital of the Central African native, nor the elephant tails of the Congo. One tail buys a slave and fifty bristles is the equivalent of $1.50.

When the United States started to mint its own coins, George Washington appointed David Rittenhouse, the eminent scientist and mathematician, as the Director. A Mint was built and coins were made on machinery operated by hand or horse power. This mint building was the first public building erected by the Federal authorities. Later it was moved to the north-west corner of Chestnut and Juniper Streets, Philadelphia. This was where the Institute's press began its operations.

The press was purchased in France in 1836, and was erected in the Mint by Merrick, Agnew, and Tyler. It coined eagles ($10 gold pieces), quarters, and cents. Its operations were so successful that similar presses were built and installed at the New Orleans, Charlotte, N. C., and Dahlonega, Ga., branch mints.

There were two reasons why this particular form of press was rendered so successful in this work. It was the first time steam had been employed in this country for a coining press. Previously, all the presses had been operated by hand or horse power. This one could be operated by water and steam, or by hand and horse power if the steam were not available.

Nor was this use of steam the only original feature of the press. Instead of the slow and cumbersome method of the screw press, it employed the toggle joint, which is a special type of link mechanism that yields great pressure. This pressure is applied in the stroke of the die. The toggle joint was a curiosity that attracted the attention of engineers when it was introduced in this press.

In the year 1858 the press was remodelled by David G. Wert. It continued in active operation until 1875, when it was withdrawn from service. This was the beginning of a new career as a showpiece. The curious public at the Centennial Exposition, the Pan-American Exposition, and the St. Louis Fair all enjoyed seeing it in operation. The Centennial Exposition Medal was made on this press. For some time after these exhibitions were over, the press languished in the obscurity of the Mint's store room from which it was reclaimed by an employee, Mr. G. B. Soley. It remained in his possession for some years, in fact until his death, and was presented to the Institute by his widow.

It would be interesting if the Museum could exhibit during next year (the 250th anniversary year of Franklin's birth) a collection of medals and coins relating to the great man whom the Institute honors. In addition to the special medals those items which would be of special interest would comprise a specimen of the first coin issued by the authority of the United States government in 1787, known as the Franklin cent because it bears Poor Richard's famous admonition of "Mind your Own Business"; an English token, dated 1794, bearing the inscription: "Payable at the Franklin Press, London." Coins issued between the year 1836 and 1876, the period during which this press was in operation, would also be acceptable exhibits.

Can any of our readers help in the collection of these coins?

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, M.D., FOUNDER
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EFFECT OF ULTRAVIOLET RADIATION ON 2-OXO-4-ARYL-5-CARBETHOXY-6-ALKYL-1,2,3,4-TETRAHYDROPYRIMIDINES

BY

HENRY A. RUTTER, JR., LILLIE O. GUSTAFSON AND WILLIAM G. BATT

In a previous publication (1) the synthesis of some 2-oxo-4-aryl-5-carbethoxy-6-trifluoromethyl-1,2,3,4-tetrahydropyrimidines was reported. As these compounds constitute an entirely new class of pyrimidines it was decided to compare them for chemical and physical properties with their known 6-methyl analogs (2). The compounds differ in that one series has an alkyl group containing hydrogen whereas the other series has an alkyl group containing fluorine. It was hoped therefore that it would be possible to evolve chemical and physical methods capable of demonstrating the fine structural differences.

In a study of 4-aryl-1,4-dihydropyridines previous investigators (3) succeeded in aromatizing the 4-(2′-nitrophenyl)-1,4-dihydropyridines by photochemical rearrangement of an ethanolic solution with a mercury lamp. As the 4-aryl-1,2,3,4-tetrahydropyrimidines are somewhat similar in structure to those pyridines, it was decided to test these pyrimidine compounds for lability of the 1,4-hydrogens under the same conditions.

Irradiation of the compounds in ethanolic solution in a quartz-bottomed dish for 48 hours with a high intensity mercury lamp produced no measurable photochemical change in 2-oxo-4-(3′,4′-diethoxyphenyl)-5-carbethoxy-6-trifluoromethyl-1,2,3,4-tetrahydropyrimidine or its 6-methyl analog; curves of the ultraviolet spectra of original and irradiated compound were identical in each instance. That the presence of a 2′- or ortho-nitro group in the 4-aryl is essential for photochemical change as in the dihydropyridines was indicated when it was found that the compound 2-oxo-4-(2′-nitrophenyl)-5-carbethoxy-6-methyl-1,2,3,4-tetrahydropyrimidine did undergo photochemical change; irradiation for 24 hours caused the yellow ethanolic solution to turn light brown (daylight) and to fluoresce dark green in the ultraviolet. Curves of the infrared spectra* of original and irradiated compound showed that considerable modification of structure had been effected

* Grateful acknowledgment is expressed to Dr. Harold C. Beachell, Department of Chemsitry, University of Delaware for arranging infrared examinations of the compounds.

by exposing the compound to ultraviolet in ethanolic solution. The 3′- and 4′-nitro compounds did not undergo photochemical change. This positional requirement of the nitro group implies that the photochemical reaction is intramolecular as it is in the dihydropyridines.

The sensitization of tetrahydropyrimidines to ultraviolet by the introduction of the 2′-nitrophenyl group is analogous to that in the dihydropyridines. This may have theoretical bearing on the photochemical reactions of naturally-occurring pyrimidines.

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NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

MINIMIZING THE TIME CONSTANT OF D-C. POWER DRIVES*

BY

BRUCE B. YOUNG

The response characteristics of a variable-speed, direct-current, electrical power transmission (such as the Ward-Leonard or Amplidyne), used to drive a primarily inertial load, depend for the most part on the principal—or "electromechanical"—time constant of the transmission. This time constant is a function of the combined inertia of the motor and load, J, the resistance of the armature circuit, R, and the voltage-speed constant of the motor, K_m, in the following way:

$$T_{em} = \frac{JR}{0.74\ K_m^2} \text{ seconds.}$$

In designing the power transmission for minimum time constant and thus most rapid response to control, it is desirable to minimize the resistance and make the voltage constant, K_m, as large as convenient. The inertia on the other hand is largely that of the load alone (if we confine our problem to power transmissions driving inertial loads), and hence is only partially under the control of the designer. To the extent that it does not conflict with other design requirements, it is desirable to minimize motor inertia.

Resistance, motor inertia and voltage constant can each be stated in terms of basic dimensions and other parameters of the transmission, so that

$$T_{em} = \frac{C_1\left[1 + \frac{\pi D}{PL\cos\theta}\right]\left[C_2 D^4 L + \frac{J_L}{n^2}\right]}{0.74\ C_3^2 B_g^2 Z_a A_a K^2 D^2 L} + \frac{R_a J}{0.74\ K_m^2},$$

where

A = cross-sectional area of conductor
B_g = air gap flux density
C_1, C_2, C_3 = constants of proportionality
D = armature diameter

* This is a condensation of a paper presented by the author at the 1955 Fall General Meeting of the American Institute of Electrical Engineers.

J_L = polar moment of inertia of load
K = ratio of pole arc to pole pitch
K_m = motor coefficient (volt/rad/sec for Ft-lb-sec units; also equal to 1.356 × lb-ft of torque per ampere)
L = armature core length
n = gear ratio
P = number of poles
R = resistance, ohms
T_{em} = electromechanical time constant, sec
Z = number of armature or pole face conductors
θ = angle between end-turn and end-face of armature core.

The first term on the right-hand side of the equation relates to the motor, the second to the generator.

In order to minimize the time constant, all the terms in the denominator should be made as large as possible and the terms in the numerator as small as possible.

B_g can be changed only by varying the number, width and depth of the slots and the diameter of the armature, since the permissible tooth-root density, and the iron stacking factor, are fairly well-fixed properties of the magnetic material selected.

The slot space availability factor depends on the slot width—being lower for narrower slots—because for a given number and arrangement of conductors, the thickness of the insulation required for the conductors is the same for all slot widths.

While B_g is increased by decreasing the slot dimensions, ZA on the other hand is increased by increasing these dimensions; hence it is necessary to find a compromise slot dimension which will give the maximum product B_g^2ZA.

Where the load is primarily inertial, the transmission will be most economical of size, weight and power consumption if the motor inertia is small compared to that of the load referred to the motor shaft (J_L/n^2). If this is so, the time constant can be reduced by *increasing* either the diameter or length of the motor armature core, up to the point where the increase in total inertia due to increase in armature dimensions cancels out the gains due to increase in the D^2L term of the denominator.

The inertia, J, can be decreased by increasing the motor speed (and the gear ratio), again provided that the motor inertia remains small compared to the inertia of the load referred to the motor shaft.

Since an increase in armature dimensions increases the size, weight and power demand of the transmission, it is necessary to weigh any reduction in time constant achieved through larger motor dimensions against the penalty which must be paid for it. Similarly, an increase in motor speed and gear ratio decreases the time constant at the expense of increased power demand, owing to the increase in power required to accelerate the motor.

Of the remaining terms, K, the ratio of pole arc to pole pitch, is dictated by other considerations. Its value cannot vary greatly from 0.65.

The inertia of the motor is minimized by choosing a long armature core of relatively small diameter, since the inertia is proportional to D^4L. The term $\pi D/LP \cos\theta$ is also minimized by using a small diamter-to-length ratio.

SUMMARY

To summarize briefly the steps a designer must take to reduce the electromechanical time constant, and thereby improve the response, of a D-C power drive:

1. Choose an armature core material having a high saturation-flux density, and capable of a high lamination-stacking factor.
2. Maintain the ratio of pole arc to pole pitch as large as possible consistent with reasonable pole flux leakage and satisfactory commutation ability.
3. Choose an armature relatively small in diameter and long, to minimize inertia and the length of mean path of conductors. The D^2L of the armature must be sufficient for proper heat dissipation.
4. Use slot insulation which will occupy the least slot space consistent with satisfactory mechanical and dielectric strength.
5. Choose slot width and depth, and number of slots, in the proportion that will give the maximum B_g^2ZA product. Choose the armature conductors accordingly.
6. Obtain a compromise between armature speed and dimensions on the one hand, and power demand, weight and size on the other.
7. Having determined motor speed and peak power, then design the generator to supply the required counter emf. plus the IR drop, and the required armature circuit current. The only restriction on the generator design is that its armature circuit resistance be as low as or lower than that of the motor. The lower it is, the lower will be the electromechanical time constant of the complete transmission.

BOOK REVIEWS

MANUFACTURE AND APPLICATION OF LUBRICATING GREASES, by C. J. Boner. 977 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $18.50.

This massive factual work in a field of great industrial importance merits the interest of all engineers. It is the most comprehensive attempt to cover the advances in lubrication technology that this reviewer has seen. It is quite apparent that the author made a special effort to organize this presentation so that it would develop logically, and be easily used as a reference. It proceeds from a discussion of definition of lubricant, importance and the many functions of lubricating greases through structure, raw materials and types of lubricating greases.

In the rather complete chapter on the structure of lubricating greases the primary components of lubricating greases are given together with the role that they play in the formation of the final product. Modern and older concepts of the colloidal state of these components are discussed. References to these modern theories are copious, and micrographs of the various fibrous structures are presented. The several factors which influence structure are discussed in detail. The important physical characteristic of greases are listed and discussed. These include such important properties as thixotropy, viscosity, syneresis among others.

This extremely valuable and interesting chapter is followed by a chapter on components that are added to improve specific properties of the grease without perceptibly altering the structure. These additives are classified, and their role described. A chapter on raw materials is detailed, lists all the important properties of the materials, and discusses the advantages and disadvantages of using each particular material. Then follow chapters on manufacturing methods, important types of greases such as aluminum, calcium, sodium lithium base greases and finally methods of testing and analyzing and methods of applying lubricating greases.

The multitudinous references, and the considerable number of excellent photographs and plots add completeness to this excellent presentation of an important field. We commend it highly. S. N. MUCHNICK

FRAMEWORK FOR ATOMIC INDUSTRY. A Commentary on the Atomic Energy Act of 1954, by Herbert S. Marks and George F. Trowbridge. 119 pages of text plus 153 pages of appendices: (A) Atomic Energy Act of 1954; (B) Atomic Energy Act of 1946; (C) Report of Joint Committee; (D) Conference Reports. 23 × 29 cm. Washington, D. C., Bureau of National Affairs, 1955. Price, $12.50.

Passage of the Atomic Energy Act of 1954 in the summer of that year constituted a declaration by Congress that it wanted the Government monopoly in the field of nuclear energy to come to an end. This change in policy by no means represented a complete withdrawal of the Government. Rather it gave permission and encouragement to the Atomic Energy Commission to liberalize its relations with private industry, so that gradually the latter can take more and more initiative in nuclear energy developments.

This book is a valuable guide to the things that industry may do. Such a guide cannot be in the form of a simple enumeration, because the Atomic Energy Act is so complicated and at times indefinite that interpretation of some of its provisions is uncertain. The authors have taken into account the past attitudes of the Atomic Energy Commission and the hearings before the Joint Congressional Committee on Atomic Energy in arriving at their interpretations. They describe the new law as a framework within which the Commission still has opportunity for considerable administrative discretion.

The changes brought about by the new law are so great that the Commission naturally has moved slowly in announcing its procedures. It was nearly a year after passage of the Act before the Commission set up a full-fledged Division of Civilian Application to match its long-standing Division of Military Application. In the meantime it had

issued drafts of some of its proposed new regulations.

Even when all these regulations have been issued, this book will be useful to a man in industry who wishes to find out what his private rights are and how he should go about exercising them in the new domain of nuclear energy activities. It is clearly written and carefully documented. A valuable feature is that areas of uncertainty are pointed out.

One important such area is that of insurance against the hazard of an atomic accident. Industry was not successful in having the law authorize the Government to underwrite the risk. The situation is now more hopeful than when this book was written; for the Insurance Study Group, made up of ten representatives of the insurance industry, issued a preliminary report in July in which it expressed hope that in most cases the insurance of privately owned nuclear facilities can be handled by the insurance industry.

In two other areas—those of patents and the dissemination of information—the situation has become more acute since this book was published. At the Geneva Conference in mid-August American business men were able to compare their opportunities in the export market with those enjoyed by their competitors abroad, especially in Britain. The comparison led to considerable dissatisfaction that they could not compete on equal terms, because of the restrictions of the American security system and the confusion concerning patent rights.

In such a rapidly developing field these areas of uncertainty will continue to change. Therefore it would be a service to industry if this book were revised annually—perhaps just after the annual Congressional hearings—so that one might continue to turn to it for unbiased interpretation of the current state of Government regulations concerning nuclear energy. JOHN S. BURLEW

TURBOBLOWERS, by A. J. Stepanoff. 377 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1955. Price, $8.00.

Written by a man who has been active in the field of turbomachines for many years, this book presents the hydrodynamic and thermodynamic theory of turboblowers. Since the mechanical design of such machines is adequately covered in the trade literature, this aspect of turboblowers has been omitted.

In order to introduce the subject properly, Dr. Stepanoff has selected pertinent topics from fluid mechanics and developed basic expressions of use in the design of turboblowers. Defining turboblowers, he then discusses the theory of centrifugal impellers and the general characteristics of turbomachinery for an incompressible fluid. Following a description of the hydraulic performance of centrifugal blowers, the thermo- and hydrodynamics of turboblowers are explained, including such special problems as the effect of humidity, Mach number and compression ratio on blower performance. Various types of blower casings are discussed along with losses in efficiency caused by leakage, disk friction and friction in the bearings and seals. Having devoted the better part of the book to theory, Dr. Stepanoff considers in the concluding chapters of his book, the design of centrifugal fans, single stage axial flow fans and blowers, high pressure multistage axial flow compressors, special problems and application of blowers and compressors, and the design of mixed flow impellers.

Generously interspersed with excellent pictures and line drawings of various turboblowers and containing many illustrative examples, *Turboblowers* should be of more than ordinary use to the engineer interested in the theoretical and practical design of turbomachinery. E. W. HAMMER

MANGANESE, METALLURGY OF THE RARE METALS, No. 3, by A. H. Sully. 305 pages, diagrams and tables, 15 × 22 cm. New York, Academic Press, Inc.; London, Butterworth Scientific Publications, 1955. Price, $6.50.

This is the third book in the series on the metallurgy of the rarer metals edited by H. M. Finniston. Manganese is generally not considered as a rare metal since it has been used as a common alloying element in steel for many years. However, manganese in a relatively pure state (99.9%) has been pro-

duced comparatively recently. In fact, the properties of manganese as reported today may need considerable revision when higher states of purity have been attained, since trace impurities in many metals affect the properties appreciably.

The pattern of the previous books "Chromium" and "Zirconium" are followed quite closely. The first forty per cent of the book covers the history and occurrence, the production and the conservation and recovery of manganese and its ores. This section of the book goes into detail only moderately, but contains material covering a wide variety of interests. For instance, the author points out that Russia is the only major steel producing country that supplies its own needs. This, coupled with the fact that manganese is one of the most important alloy additions to steel, makes it easy to understand why this metal is of such strategic importance.

Chapter five is an excellent compilation of the properties of manganese. It includes a detailed discussion of the crystal structure in its four allotropic forms. The electrical and magnetic properties are summarized and many of the physical constants are tabulated. Some of the thermodynamic properties are included.

The mechanical properties were covered in one page. It is unfortunate that more data were not presented on this subject. The author explains this omission by pointing out that the existing data are not reliable because of the difficulty of obtaining sound test pieces free from cracks. However, poor data are better than none and can be especially valuable to those who are trying to improve the existing information.

One of the most useful sections of the book to the physical metallurgist is probably the chapter on the constitution and properties of manganese alloys. The phase diagrams of twenty-four binary systems are given. In addition, the author discusses the known data on twenty other systems. Considerable detail is given in those cases where the equilibrium diagrams are controversial. For example, three different versions of the Mn-Cu system are shown and discussed. Thirteen pages are devoted to the constitution and properties of the Mn-Cu alloys. Twenty-three ternary systems are also discussed in this section.

The book is well-referenced and is up to date. It maintains the high standards set by the previous texts in this series.

R. L. SMITH

ADVANCES IN GEOPHYSICS, Volume II, edited by H. E. Landsberg. 286 pages, illustrations, 16 × 24 cm. New York, Academic Press, Inc., 1955. Price, $7.50.

This is the second number of the *Advances in Geophysics* series and, like the first, it cuts across the subfields and culls from other disciplines techniques which profoundly affect present day results.

The book is divided into five sections of which the first two deal with weather. The first chapter dealing with advances in radar weather develops the application of this new medium to weather and illustrates the various weather phenomena with pictures of the PPI screen. This is a wide open field possessing so much potentiality that it may prove a major investigative field of the immediate future. The authors have performed a magnificent service in the interpretation of this field.

The second chapter deals with methods of objective weather forecasting. Here is stressed the desirability of objectivity in the interpretation of data for the author feels that only in that manner can forecasters come close to the elusive goal: accuracy. From the exposition set forth it seems that the human equation is the one most likely to be at fault. He hopes to remove it by setting forth weather aspects as impersonal mathematical formulae and equations.

Section three deals with Wind Generated Gravity Waves. It attempts to analyze, with the aid of new theoretical developments, the generation, propagation and refraction of such waves. Unfortunately the subject seems to be ill-defined and the author is trying to set down a basic set of rules which may lead to a better understanding of this phenomenon.

The fourth section will engender tremendous interest on the part of the non-specialist. Geological Chronometry by Radioactive Methods has been used by top drawer scientists in dating events and objects of the past. With the advent of the atomic age and the

new disciplines which evolved, many investigators in the geochronometry field have been given a powerful probe in determining ages. The various methods in use are delineated and the possible errors set forth. Here is contained the most complete exposition of this field that this reviewer has seen. It is written by a master craftsman and merits complete confidence.

The last section deals with the instruments for recording earthquakes and associated tools. It is a compendium of seismological instrumentation and anyone contemplating the installation or maintenance of such a station should consult this work. Hugh Benioff is a recognized master in this field and his survey is excellent and complete.

Advances in Geophysics is an excellent exposition of some facets of geophysics though this reviewer found that in some places the writing exceedingly specialized. It appears that a time will come when the editor must decide whether these volumes are for the general scientist or the specialist in the field. Right now the books are tending toward the latter and it would be indeed unfortunate if the writing became so abstruse that the restricted sales of the books precluded the completion of the series contemplated.

I. M. LEVITT

ANALYSIS OF FEEDBACK CONTROL SYSTEMS, by Robert A. Burns and Robert M. Saunders. 383 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Company, Inc., 1955. Price, $7.50.

For more than a century, man has been confronted with the necessity of providing various degrees of automatic control over the machines he has developed. During this period, great advances have been made in the understanding of the physical aspects of these machines; only minor considerations were extended to the control problem. Much of the control engineering was left to the "cut-and try" approach. However, conditions of the past twenty-one years have all but eliminated this deficiency. The greatest share of the credit for the present state of the art is given to H. L. Hayen's paper of 1934, and the violent rapidity with which the needs for more accurate control were made manifest during the Second World War.

Confronted with the necessity of providing a means of establishing conformal system parameters prior to the designing of system components, a concerted vigor has been expended on the dynamics by many quarters. These efforts have resulted in the development of several predominate methods of analysis. Each has been exploited along the avenues presented by the representative problems. At the same time, many people were devoting efforts to a systematic verification of the mathematical methods. In accounting for an absence of a discrete concurrence between model data and the mathematical treatment, new and more concise methods of analysis have been established by repeated correlation of the evolved analysis with the model data.

Through this mass of recorded information has come many profound works on the associated subjects of: servomechanisms, "feedback" control and automatic control. Unlike the many works which have preceded it, this most recent addition does not advocate any new concepts or processes. Rather it represents a logical step in the direction of solidifying the efforts of the many contributors to the state of the art. This volume would appear to follow the time proven "elements" approach. That is, it does not exist as a random collection of articles on the subject of Automatic Control. Rather, each of the components which might be found in a system is treated individually and then collectively in the loop configuration.

After the introduction, all the remaining chapters in this book fit into one of two sections. The first group treats the dynamics and transfer functions of the components; analysis of feedback control systems consumes the second part of the book. Each section is capable of individual treatment, and, as such, contains its own background and builds up its own presentation.

It is interesting to note that the authors include a great variety of components which could be classified under the following headings: mechanical, pneumatic, hydraulic, magnetic and electronic. Although many modes of system analysis are presented, use of the frequency-response methods is emphasized as being the best suited for general considerations. Utility is made of this proven ap-

proach not only for analysis, but also as the underlying framework for synthesis.

The authors have compiled in this book a natural transition from the realm of statics to that of system dynamics. This work is further enhanced by the unified method of analysis which is placed on a broad conceptional basis. As such, relative ease of comprehension is extended to all. At the same time, it holds great utility as a reference for both individual transfer functions and an exhaustive survey of the modes of system analysis. T. J. BOWDEN

BASIC LUBRICATION PRACTICE, by Allen F. Brewer. 286 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1955. Price, $6.75.

In recent years great strides in research and development have been experienced in the field of lubrication. The intensive effort in this field has been primarily due to the extreme demands placed on our modern machinery. High speed and load and in many cases high ambient temperatures are the necessary conditions for moving parts where peak over-all efficiency is to be obtained. Sound and advanced lubrication practice is vital.

This book is one of the most complete compilations describing the proper lubricants and methods for lubricating components of machines. Not only does it cover thrust and journal bearings and gears, but also includes cams, chains, wire rope, couplings and seals. Aside from chapters devoted to the lubrication of specific machine components, the general subjects of lubricants and bearing materials are treated separately. Lubricants are divided into oils, greases, synthetics and solids and are described in tables and graphs as to their characteristics and recommended use. Bearing materials are treated in a similar manner and cover the range from wood—the earliest known—to the laminated synthetics and nylon.

Because of the author's extensive knowledge in the field he was able to devote a portion of the book to the lubrication of complete machine systems. Chapters are devoted to such systems as steam turbines, internal combustion engines, electric motors and air and refrigeration compressors. In all cases, the text is supplemented with many photographs and charts.

Although this book is mainly designed for maintenance engineers and machine operators, it could serve as an excellent text for college or trade school students for studying practical lubrication. One example of its value to the student is the many photographs showing various stages of bearing and gear failures that normally would be observed in the field.

In general, this book when used in conjunction with a text on bearing theory and design would provide the basic "tools" required for the solution of unconventional as well as the more conventional lubrication problems. STANLEY ABRAMOVITZ

INFORMATION PROCESSING EQUIPMENT, edited by M. P. Doss. 270 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1955. Price, $8.75.

Science advances so rapidly today that many reports get overlooked in the shuffle. To guard against this, authors wish to know how to prepare reports and illustrate them rapidly and cheaply. Speakers who want to use the lecturing equipment and techniques advantageously will find an up to date chapter on projection equipment and slide preparation. The experimenter needs to use the best methods for collecting and processing data. Maybe the "archaic lead pencil" will serve. However, some types of data can be obtained only by the more modern recording techniques explained in the book. Once obtained numerical data need to be processed. The various computers from simple slide rules and calculators to "giant brains" are compared in ability and cost. Some of these same machines can also be used for filing and sorting information and reports. While the two chapters dealing with these business and scientific calculating machines of course can be only a brief survey, they are supplemented by well selected and extensive bibliographies which will aid persons interested in this phase of information processing.

Reports can be prepared using special typewriters, hectograph machines, facsimile, xerography, or commercial printing machinery. The areas in which each technique is most valuable are pointed out. Illustrations

may be copies of existing material obtained by photographic or xerographic methods or they may be originals obtained in any of several ways compared in the book. Once the report is finished, micro-copying may be used to eliminate the bulky storage problem in large libraries. Micro-copying, projection, and filing equipment are compared for cost and use.

Before reports can be written, data must be obtained. Automatic methods can gather more data and speed experiments. Sometimes this is the only way to obtain the data. The ways in which photography can be used are summarized. Tape, disc, and wire recording equipment each has its own advantages. Using the voice recorder eliminates the assistant data recorder, speeds the experiment and allows many additional observations to be logged. In some cases audio tracks on film may be advantageous.

A problem every engineer, scientist, and physician faces today is how to find information. Punched cards such as the E-Z Sort, Keysort and others are suitable for small personal files. Business machines and large digital computers can be used for larger central files. Unfortunately the authors can not provide the final answer for a method for searching the literature and obtaining all the available references on the subject because techniques and equipment have not been perfected.

This text is a well written comparison of equipment and techniques and is supplemented by an extensive bibliography at the end of each chapter. It includes figures on the costs of machines and materials. Since most engineers and scientists are concerned with some phase of information processing they will find this a very useful reference book. EUGENE A. MECHLER

MACHINE TRANSLATION OF LANGUAGES, edited by William N. Locke and A. Donald Booth. 243 pages, 16 × 24 cm. Cambridge (Mass.), The Technology Press; New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1955. Price, $6.00.

The application of large scale digital computers to the solution of problems in science and business is today commonplace. Less so, however, is the application of such machines to the field of communications, in the broad sense, and to the field of automatic machine translation of languages in particular. The use of such digital techniques is of interest to the members of a wide gamut of scientific and social disciplines, such as sociologists, linguists, philosophers, mathematicians, electronic scientists, and philologists, and masterminds of international propaganda.

Written by seventeen authorities drawn from the above fields of endeavor, this first definitive book on the subject of machine translation brings to the reader the most recent ideas of their authors through the medium of fourteen essays. These are devoted to such topics as: the design of an automatic dictionary; problems of the "word"; an operational analysis of Russian; input and storage devices; idioms and syntax, and model English, which was the most intriguing essay.

Several of the individual essays represent extensions of the research of their authors, since they were initially publicized in the first symposium on this subject held at Massachusetts Institute of Technology in 1952. This gathering occurred three years after Dr. Warren Weaver, Secretary of the Rockefeller Foundation, privately circulated to a small group a memorandum entitled "Translation," which memorandum is reprinted in this book. That a sufficient number of minds were stimulated to consider the problem seriously is evidenced by the number of persons now engaged in research in various aspects of the basic subject.

To date no extensive application has been made of digital techniques to machine translation, although several experiments have been tried with varied degrees of success. The major stumbling block is the problem of quantity and capacity of storage devices, but these problems are bound to disappear in the near future. The costs of machine translation, however, may raise other problems to limit its application. S. CHARP

SOLVENTS MANUAL, compiled and edited by C. Marsden. 429 pages, 16 × 25 cm. Houston, Elsevier Press, 1954. Price, $12.95.

This is entirely a reference book and should be of value to workers in industries that require comprehensive information about many solvents. After laying a common

ground for understanding by defining the terms used in presenting information about the solvents, all pertinent information, physical characteristics and properties are listed. These include characteristics not found in handbooks such as physiological properties, storage and handling problems and manufactures. The various industrial grades are listed.

The appendixes contain information such as chemical classification of solvents, methods of volumetric determination of various solvents, conversion tables and other pertinent facts. A removable solubility chart of resins, wax, fats, etc., is included. It is certainly a book that will not remain untouched for any extended period.

S. N. MUCHNICK

GAS TURBINE. VOLUME 1. CYCLES AND PERFORMANCE ESTIMATION, by James Hodge. 329 pages, diagrams, 16 × 25 cm. New York, Academic Press Inc., 1955. Price, $9.00.

This is the first volume of a series on Gas Turbine theory. It covers performance characteristics for compressors, gas turbines and over-all plant combinations with combustion chambers. These include a straightforward analysis of thermal efficiency, specific output and other factors for estimating performance for various cycles and combinations of power units. The definition of efficiencies are relative to the isentropic temperature rise, to the actual irreversible adiabatic temperature rise at a given pressure ratio for the compressor, and the irreversible adiabatic temperature drop divided by the isentropic temperature at a given pressure drop for the turbine. Gas turbine analysis permits the direct application of temperature changes in place of enthalpy changes, using approximate values of the specific heats. Using assumed compressor and turbine efficiencies, which account for the losses, pressure ratios are established in terms of the actual temperatures. This method is compared with polytropic efficiencies.

The author's procedure of cycle analysis is effective in bringing out basic characteristics of gas turbine cycles. Thus the effect of heat exchangers on efficiency is analyzed, showing that by reducing the temperature rise through the compressor, the heat exchanger has a marked effect and in fact with heat exchangers the optimum pressure ratio can be reduced. On the other hand, when the compressor temperature rise equals the turbine exhaust temperature, the heat exchanger has no effect.

The author includes the analysis of jet propulsion cycles. The effect of maximum cycle temperature must be balanced with propulsion efficiency, and with corrections for room pressure ratios. Such methods as the optimum compressor temperature rise for maximum thrust are developed.

Chapters 1 and 2 give basic data on methods of cycle calculations with analysis for shaft power and jet propulsion types. Chapter 3 is devoted to the performance for shaft power only, while Chapter 4 covers aircraft engine cycles. Chapter 5 gives various applications of the gas turbine cycle, as for gas turbine blowing cycles, and in steam cycles as in the Velox boiler.

The closing chapter on Part Load Performance is of particular importance and the author has given a good survey for the estimation of performance away from the design point.

The approach used throughout the book in terms of temperature changes and temperature ratios as principal coordinates for gas turbine performance has been extended beyond that found in the literature. The question arises, however, whether a more basic approach on the changes in thermodynamic properties of the flow through the compressor, combustion chamber, turbine and jet, taking into account a more physical picture of the energy transfers at turbine, compressor, etc., should not be initially introduced. While some such approach can be made later, it is to be regretted that this method was not introduced at the outset.

RUPEN EKSERGIAN

GAS TURBINE. VOLUME 2. SOME FUNDAMENTALS OF COMBUSTION, by D. B. Spalding. 250 pages, illustrations, 16 × 25 cm. New York, Academic Press, Inc., 1955. Price, $7.50.

This volume is essentially a general analysis of combustion. Chapter 1 covers a brief review of thermodynamic principles and energy balances. Chapter 2, on fluid flow, considers compressible flow with chemical

reaction. Preliminary ideas on flame shape and velocity distributions for flow in a duct are included.

Chapters 3 and 4 offer an interesting contribution on heat and mass transfer, covering such matters as flow in ducts and packed columns, simultaneous heat and mass transfer. The heat and mass transfer with chemical reaction is extended to the phenomena of the combustion of gas jets including the burning rates of solid and liquid fuels. Chapter 4 concludes with a study of surface combustion, covering composition and temperature profiles adjacent to combustion surfaces.

Chapter 5 is a brief survey of chemical effects in combustion as with pre-mixed gases, flame propagation and quenching effects. The role of turbulence, flame stabilization by recirculation, spontaneous ignition and partially mixed flames are discussed in a very comprehensive manner.

The last chapter on Design Methods is of particular use to designers. It gives a clear picture for basic proportions and arrangement of baffle grids in combustion chambers.

This volume can stand on its own as a basic contribution to combustion phenomena. It probably is the most important single problem in a successful gas turbine design. The author has also included an extensive reference section of real value for the research worker in combustion phenomena.

RUPEN EKSERGIAN

THERMODYNAMICS, by Ray L. Sweigert and Mario J. Goglia. 355 pages, 16 × 24 cm. New York, The Ronald Press Company, 1955. Price, $6.50.

This book is designed as a textbook for Junior and Senior students in engineering. The topics covered are the First Law of Thermodynamics including applications, the Second Law, followed by a discussion of Properties and Perfect Gases. The topic on Gas Dynamics, pertains to the flow of gases. Chapter 7 includes the equation of state of vapors and gases with some discussion on thermodynamic properties. This is followed by a chapter on Perfect Gas Mixtures and Vapor Gas Mixtures.

The chapter on Thermodynamics and Chemistry is an interesting contribution not usually found in the more orthodox engineering texts. A very brief discussion is given to compressor and turbine theory with applications to thermodynamics of fluid flow.

The authors' development of the Second Law could be improved, and the discussion on Availability and Irreversibility should be coordinated with this development. The analysis of fluid flow and turbine theory is well presented. RUPEN EKSERGIAN

BOOK NOTES

SHELL THEORY OF THE NUCLEUS, by Eugene Feenberg. 211 pages, 18 × 25 cm. Princeton, Princeton University Press, 1955. Price, $4.00 (paper).

This is the third in the series of books titled "Investigations in Physics." It is an authoritative monograph inasmuch as Dr. Feenberg has made numerous contributions to nuclear shell theory.

The subject is introduced in the first chapter by a discussion of the origins of the theory, namely the experimental findings of magic numbers. In the second chapter the author formulates shell models. Chapters three to nine are concerned with the application of shell theory to the interpretation of magnetic dipole and electric quadrapole moments, isomeric transitions, beta transitions, superallowed transitions, and spins and parities. In the final chapter the author discusses nuclear force models and the question of why the shell model works so well.

A thorough familiarity with nuclear physics and quantum mechanics is required for a proper understanding of the contents. There is a name index and a very brief subject index. Since there is a limited audience for such a book, costs have been held to a minimum by offset printing and paper binding.

PLASTICS TOOLING, by M. Riley. 123 pages, illustrations, 12 × 18 cm. New York, Reinhold Publishing Corp., 1955. Price, $2.50.

One of the new Pilot Book Series, designed to cover new developments as they occur, this small volume describes briefly the new

science (rather, art) of plastics tooling. It is a simple report, based on what has already been published and on interviews which the author has had with people actively engaged in plastics tooling. Ten chapters, adequately illustrated, cover an introduction to the general subject, the resins used for plastics tooling, the fabrication of the tools, assembly fixtures, die models, etc. The tenth chapter also covers prospects for the future. An index and a selected bibliography of ten references (all 1954) complete this informative report.

MATERIALS FOR NUCLEAR POWER REACTORS, by Henry H. Hausner and Stanley B. Roboff. 224 pages, 12 × 18 cm. New York, Reinhold Publishing Corp., 1955. Price, $3.50.

Another welcome addition to Reinhold's Pilot Book Series, this latest book should be of interest not only to those directly concerned with nuclear power reactors, but also to the layman whose future will be influenced greatly by this fast-growing industry.

The present work summarizes, in readily understood terms, the more important materials used in nuclear reactors—for shielding, for cladding the fuel elements, for moderators and reflectors. The book may serve as a guide for designers of nuclear reactors.

In addition to describing the basic parts and materials of reactors, special problems both in construction and operation are outlined; also, nuclear fuels and coolants are treated.

A valuable (to the layman) glossary of terms occupies 12 pages; there are two pages of references covering the years 1947 through 1954; and an index completes the book.

PUBLICATIONS RECEIVED

THE BRITISH JOURNAL PHOTOGRAPHIC ALMANAC 1955. 640 pages, illustrations, 13 × 19 cm. London, Henry Greenwood & Co., Ltd., 1955. Price, $2.00.

FUNDAMENTALS OF GENERAL CHEMISTRY, by C. H. Sorum. 588 pages, diagrams, 16 × 24 cm. New York, Prentice-Hall, Inc., 1955. Price, $8.65.

QUANTUM MECHANICS, by Leonard I. Schiff. Second edition, 417 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1955. Price, $6.50.

COLLECTED WORKS OF MARTIN ANDRÉ ROSANOFF. v.p., 15 × 23 cm. New York, Galois Institute of Mathematics, no date. Price, $10.00; $5.00 (to members of Galois Inst.).

DICTIONARY OF TERMS USED IN THE THEORY AND PRACTICE OF MECHANICAL ENGINEERING, originally compiled by J. G. Horner. Seventh edition, 417 pages, 13 × 19 cm. New York, Philosophical Library, 1955. Price, $6.50.

ELECTRO-TECHNOLOGY, by M. G. Say. 167 pages, diagrams, 15 × 22 cm. New York, Philosophical Library, 1955. Price, $6.00.

GENERAL PRINCIPLES OF GEOLOGY, by J. F. Kirkaldy. 327 pages, plates, 16 × 24 cm. New York, Philosophical Library, 1955. Price $6.00.

EXPERIMENTAL PSYCHOLOGY, edited by B. A. Farrell. 66 pages, 13 × 19 cm. New York, Philosophical Library, 1955. Price, $2.75.

MICROBIOLOGY, AN INTRODUCTION, by Ernest A. Gray. 175 pages, 13 × 19 cm. New York, Philosophical Library, 1955. Price, $3.75.

LAPLACE TRANSFORMS FOR ELECTRICAL ENGINEERS, by B. J. Starkey. Reprint, 279 pages, 15 × 22 cm. New York, Philosophical Library, 1955. Price, $10.00.

THE ELEMENTS OF THE THEORY OF REAL FUNCTIONS, by J. E. Littlewood. Third edition, 71 pages, 14 × 20 cm. New York, Dover Publications, Inc., 1955. Price: $1.35 (paper); $2.85 (cloth).

CURRENT TOPICS

Plans for Earth Satellite Announced.—Plans for the construction of a small, unmanned, earth-circling satellite vehicle to be used for basic scientific observations during the forthcoming International Geophysical Year have been announced by Detlev W. Bronk, President of the National Academy of Sciences, and Alan T. Waterman, Director of the National Science Foundation. The project which is entirely scientific in nature, will be sponsored by these two organizations as part of the United States program of participation in the International Geophysical Year. Technical advice and assistance will be provided by the scientists of the Department of Defense who have long been engaged in research on the upper atmosphere. The Department of Defense will provide the required equipment and facilities for launching the satellite.

The program for such a vehicle was stimulated by a resolution passed by the Special Committee for the International Geophysical Year (French abbreviation CSAGI) at its Rome Meeting in October, 1954. It is planned that the developmental work be completed in time for a successful launching during the International Geophysical Year, which is a period set aside during 1957 and 1958 for world-wide observations in the fields of the earth sciences by some 40 nations.

In its resolution, the Committee stressed the great importance of observations of extraterrestrial radiations and geophysical phenomena for extended periods of time. The Committee's recommendation urged that participating nations give consideration to the construction of small satellite vehicles, instrumented to provide such data as may be feasible from outside the earth's atmosphere.

The atmosphere of the earth acts as a huge shield against many of the types of radiation and objects that are found in outer space. It protects the earth from things which are known to be or might be harmful to human life, such as excessive ultra-violet radiation, cosmic rays, and those solid particles known as meteorites. At the same time, however, it deprives man of the opportunity to observe many of the things that could contribute to a better understanding of the universe. In order to acquire data that are presently unobtainable, it is most important that scientists be able to place instruments outside the earth's atmosphere in such a way that they can make continuing records of the various properties about which information is desired. In the past vertical rocket flights to extreme altitudes have provided some of the desired information, but such flights are limited to very short periods of time. Only by the use of a satellite can sustained observations in both space and time be achieved. Such observations will also indicate the conditions that would have to be met and the difficulties that would have to be overcome, if the day comes when man goes beyond the earth's atmosphere in his travels.

The satellite itself will orbit around the earth for a period of days, gradually circling back into the upper atmosphere where it will eventually disintegrate harmlessly.

The National Academy of Sciences-National Research Council, which represents the interests of United States

scientists in the International Council of Scientific Unions, is responsible for development of the scientific program to be undertaken by this country during the International Geophysical Year. The Academy-Research Council established the U. S. National Committee for the International Geophysical Year, composed of leading American scientists, to plan and carry out the International Geophysical Year program. This committee is assisted by twelve technical panels dealing with the various phases of the program of which the earth satellite is a part.

Special Federal appropriations being made in support of United States participation in the International Geophysical Year are being administered by the National Science Foundation which is coordinating Government interests in the over-all program.

Image Amplifier Tube Uses Gas Which Traps Electrons.—The image amplifier tube employs perfluoro-cyclic ether, one of the newest insulating materials in use in the electrical industry. This gas insulator, adapted by the Westinghouse Electric Corporation's X-Ray Division in Baltimore, is composed of carbon, fluorine and oxygen. The gas is used inside the tube where the electrical connections are made.

The image amplifier tube is basically a light amplifier tube. It takes very dim fluoroscopic images at one end of the tube and transforms them into bright light images (200 to 400 times) at the other end. Therefore, any bright light outside the tube formed by corona would shine inside the tube and also be brightened. This would blot out the fluoroscopic image which the physician is trying to observe.

By excluding the air from within the housing and replacing it with a suitable insulating gas no visible corona is formed until voltages considerably above 30,000 are reached. A much larger housing, filling the air space with oil, making large rounded smooth connections are other methods used to prevent corona. However, these methods are impractical, because both weight and size of the unit would have to be increased significantly.

The gas is introduced into the housing in a simple manner. First, most of the air is replaced by nitrogen gas. This is done by flushing the nitrogen through the housing. Then two cubic centimeters of liquid perfluorocyclic ether are injected into the housing, through a rubber disc, by means of a hypodermic needle. This liquid evaporates in the nitrogen, and the resulting gas mixture has about $2\frac{1}{2}$ times the electrical strength of air.

Perfluoro-cyclic ether is an electronegative material, therefore, it tries to capture any free electrons moving about in the gas. Since corona is formed by the movement of free electrons and ions, higher voltages are necessary to form corona in this gas.

Outdoor Thermostat Keeps Homes Comfortable.—The general Electric Company has announced the development of an automatic outdoor thermostat control which is expected to cost consumers 75 per cent less than existing automatic systems. In operation, the outdoor control tips off the indoor thermostat to changing weather conditions. This helps prevent indoor temperatures from dipping with sudden outside changes.

Ordinarily, G-E engineers explained, a drop in outside temperature means that the homeowner is required to adjust his indoor thermostat upward to compensate for heat loss through the colder walls. The new system does this automatically, eliminating manual adjustment on the part of the homeowner.

Heart of the outdoor thermostat is a

small wafer of a peculiar carbon compound that senses weather changes and then sends a minute electrical signal to the indoor thermostat. Thus warned, the indoor thermostat turns on the furnace to cope with lower temperatures.

General Electric's Appliance Control Department has announced plans for marketing the device through furnace manufacturers and distributors. It is expected to be available by January, 1956.

Multiple-Range Strip Chart Recorder.—A multiple-range strip chart recorder, which automatically switches between a series of full scale ranges of 0–110, 100–210 and on up to 900–1010 millivolts, is now being produced by Applied Physics Corporation, Pasadena, California. The Model 39 multiple-range recorder is said to be particularly useful for applications, such as measuring small voltages or currents with an electrometer, where e.m.f. values must be recorded with high accuracy over a wide range of values. Range switching is performed by limit switches at the ends of the recorder slidewire, so that a higher or lower range is automatically selected when the recorder pen reaches either end of the scale.

Isotopes Identify Petroleum Oils in Pipelines.—Scientists at Britain's atomic energy research establishment in Harwell, Berkshire, have worked out an easy method of identifying different types of petroleum oils, as the oils flow through pipelines. The method is designed in such a way as to enable the interface (the point at which two oils mix) between two oils or fuels to be located or followed during transmission along the same pipe-line.

So far the laboratory tests made at Harwell have shown that the method can be used to differentiate between premium or commercial grade fuels by a five per cent change in the reading between the two types. When commercial fuels and paraffin are examined there is a further change of ten per cent.

Asphalt Against Atomic Fall-Out?—Public-safety officials are looking to asphalt, a petroleum product, as a possible protection against radioactive fall-out after atomic tests. It is believed that the asphalt may hold together the very light particles of soil so that fewer of them will blow off in the mushroom cloud when the nuclear device is discharged at the test site. The use of an asphalt base may result in forming lumps of soil which will either stay on the ground or fall out quickly near the range, thus giving greater protection to the population in fall-out areas. If the tests prove successful, asphalt may soon perform a new service, along with fuels, lubricants, and oil chemicals, in petroleum's alliance with the atom for America's defense.

Quick, Watson, the Blots!—Oil company engineers have perfected a motor oil test that can be performed by the driver, and which tells in minutes whether the oil in an engine needs changing. The test is based on the behavior of oil blots on filter paper, and a chemical color change that indicates the degree of acidity or alkalinity. The test not only shows when the oil needs changing, but also warns of cracked pistons, antifreeze leakage, and similar troubles.

Heating Device Aids Oil Wells.—Nearly 20 per cent of the nation's oil resources—hitherto beyond reach because of the earth's paraffin barrier—can now be tapped.

A heating device, manufactured jointly by N. W. Curson Associates

of California and the General Electric Company, literally melts its way through the paraffin barrier found in most low-producing wells, and opens the way for consistent pumping.

The paraffin barrier, as it is known to geologists, is the gummy substance formed from the heavy, sludgy crude oil found in many wells.

The 18-ft. long heater assembly includes six 80-in. Calrod * heater units, manufactured by G.E.'s Industrial Heating Department. The heater is sealed in oil-tight pipes which are enclosed in a rigid conduit and then lowered into the well shaft.

The California firm has reported that the production of a West Coast well increased from 5 to 45 barrels a day, following use of the heater.

According to the Curson organization and G.E., the new heater is expected to boost oil output some 900 per cent in low-producing wells.

Nut Holding Tension-Grip Nut Drivers.—Television and radio servicemen no longer need fuss or fumble with small nuts or hex screws—International Resistance Company, Philadelphia, has made available a Tension-Grip Nut Holding Nut Driver. Ideal for close work starting or removing in television and radio servicing, the Tension-Grip Nut Driver has a tempered steel band that grips the nut or hexagon head screw automatically as it enters the socket; it will not drop it. The "tension-grip" has a spring action and is non-magnetic; handles brass and stainless steel nuts as well as regular types. The nut driver shaft is securely set in a shock-proof plastic handle, and is available in six popular sizes: $\frac{1}{4}''$, $\frac{5}{16}''$, $\frac{11}{32}''$, $\frac{3}{8}''$, $\frac{7}{16}''$, $\frac{1}{2}''$.

* Registered trademark of General Electric Co.

Thumb-Sized Motor for Aircraft Applications.—A tiny motor, no larger than a man's thumb has been developed by General Electric for a wide variety of aircraft applications. Manufactured by the company's Instrument Department at Lynn, Mass., the little motor was originally designed for use in the gyro component of G-E's MA-1 compass-controlled directional gyro-compass system. It is believed to be the smallest motor currently used on aircraft, and G-E engineers predict that designers of aircraft instruments and servo systems will find it can be adapted to many other uses.

Rated at 26 volts and weighing just 1.2 oz., it is $\frac{5}{8}$ in. in diameter and 1.2 in. long. Its no-load speed is 21,000 rpm., and it has exceptionally high acceleration, stall torque and efficiency for a motor of its size and type. It is designed to withstand ambient temperatures in the range of $-55°$ C. to $+90°$ C. when operated as a control motor, and is built to provide satisfactory performance at altitudes up to 60,000 ft.

This two-phase, low inertia servo motor may be used in a wide variety of circuits to make possible accurate null-method measurements as well as to provide the power required to initiate control.

In one of the simplest of these circuits, one phase of the motor is continuously excited from the 400-cycle line, while the other is fed from the output of a servo amplifier. The necessary 90-degree phase shift between the input to the fixed phase and the variable phase may be introduced in the error signal circuit, in the amplifier, or in the fixed-phase circuit. In this way, the error signal from a synchro-control transformer may be amplified and used to initiate control through the servo motor.

The motor may also be used as a

power failure indicator or as a power-operated brake, as is the case in G-E's MA-1 compass systems. When the pilot switches off his gyro, the tiny motor, which has been stalled while the gyro was operating, actuates a braking gear, preventing the gyro from spinning and tumbling when it has no work to do. This affords protection against damage and assures high-level performance.

Stereophonic Music for Homes.—The world's first stereophonic music system for home use was unveiled by the Ampex Corporation at the annual convention of the National Association of Music Merchants at the Palmer House. Described by Phillip L. Gundy, manager of Ampex's audio division, as "the next logical step" in the realistic reproduction of recorded music, the system is based on the Ampex Model 612, a new tape phonograph.

For stereophonic reproduction, the Model 612 plays each of two separately recorded sound tracks from a single tape through two separate amplifier-loudspeaker systems. Thus, music originating on the left side of an orchestra is reproduced through the left hand loudspeaker, and music from the orchestra's right comes through the right hand loudspeaker. The result is a sense of direction and depth never before achieved in a home music system.

In addition to playing commercially recorded stereophonic tapes, the new phonograph also can reproduce standard tape recordings, whether recorded by a record manufacturer or at home. Accommodation is provided for both full track tapes, in which the signal covers the full width of the tape, and half track tapes, in which two different recordings each using only half the tape width, are recorded in opposite directions. Half track tapes have the advantage of storing twice as much music on a single reel of tape.

Stereophonic tape recordings are currently being marketed by most of the manufacturers of tape recordings. More selections are being released monthly. In addition major record manufacturers are recording all sessions stereophonically, looking forward to the day when they will release more of their libraries on tape as well as disc, Gundy said.

To enable purchasers to take advantage of amplifiers and loudspeakers which they already possess, the Model 612 tape phonograph is sold without audio amplifiers or speakers. Thus, it can be connected into existing high fidelity systems. An additional amplifier and speaker must be provided if sterophonic sound is to be reproduced.

Maximum realism is achieved if both tracks of stereophonic recordings are played through identical amplifiers and loudspeakers. Thus, a complete Ampex stereophonic system not only includes the Model 612 phonograph but also two Model 630 amplifier-speaker systems.

Also being shown for the first time at the convention, the Model 630 is a combination amplifier-speaker unit. It incorporates a 20-watt audio amplifier, a loud-speaker embodying design techniques never used before and an acoustically matched enclosure. Separate controls are provided for bass and treble tone ranges. A selector switch enables it to play from phonograph, radio, TV receiver or tape reproducer. Frequency range is extended considerably, even over the earlier Ampex Model 620 amplifier-speaker.

Old Searchlight Creates Artificial Moonlight.—The old 60-in. searchlight of World War II has been adapted for a new role by the Illumination Section of the Corps of Engineers' Re-

search and Development Laboratories, Fort Belvoir, Virginia. Supplanted by radar for aircraft detection, it has been transformed into a self-contained mobile unit to supply battlefield illumination or "artificial moonlight" for combat operations.

Originally designed for transport in two trailers towed by a pair of 2½-ton trucks, the entire unit has been mounted on a single truck of the same size. The old method of transporting the unit was adequate for anti-aircraft units emplaced in rear areas, sometimes at the same location for days. Mounting the entire unit on one truck, however, became necessary to permit the rapid and frequent movement required for front-line battlefield illumination.

The new truck is equipped with sides which can be let down to form a platform for the operator. The engine generator set which powers the unit is mounted directly behind the cab of the truck.

Army units achieved much success in Korea by locating the searchlights 5 to 10 thousand yards behind the area they wanted illuminated and directing the beams at a low elevation. Light, scattered from the beams, provided illumination equivalent to an almost full moon over an area 800 yd. wide and 1500 yd. long. Greater width but less illumination was obtained when they defocussed the beam.

Since it is directional and can be controlled, the light gives our combat troops a considerable advantage over the enemy.

Searchlights can also be used to illuminate job sites and rear area installations. They may also serve as beacons.

Multi-purpose Crate and Carton Clamp.—A wide variety of carton and crate handling jobs can now be accomplished with a versatile, new hydraulic clamp with rotator and side shifter developed by The Yale & Towne Manufacturing Company. This new device eliminates the need for several attachments where different box handling assignments pose a problem. The clamp handles with ease crates, capped cartons and regular cartons in a great range of sizes.

Basically a vertical clamp, the distance between the bottom forks and the top plate can be hydraulically adjusted from 26 to 76 in. This allows for the handling of crates and regular cartons in a height range of from 28 to 74 in.

The stationary forks are covered with an abrasive coating and slippage from the top plate is eliminated by adjustable spikes. This top plate also can be equipped with a small lip to fit between the slats of crates for greater load stability while rotating the clamp.

Capped cartons in a range from 32 to 78 in. in height can be handled with this versatile device. A blade on the top plate slides under the lip of cap and hinged stops are swung into place on the lower forks in this type of handling.

In any kind of handling operation the clamp can be shifted four inches from center in either direction for quick, effortless positioning of the load. When handling regular cartons or crates, the clamp can be rotated through a full, 360-deg. circle enabling either vertical or horizontal storage easily.

Loads handled with the clamp can be lifted to the full truck fork height in any position.

Mounted on a truck of 1500-lb. capacity this attachment has a lifting capacity of 1000 lb. and a rotating capacity of 500 lb.

Klystron Survives Record-Breaking Navy Rocket Flight.—Surviving the impact following a record-breaking altitude flight of 158 miles in the after-

body of the Viking No. 11 rocket, a Varian Associates Model V-55 reflex klystron was recovered in operating condition. This information was released to Varian Associates by the U. S. Navy Chief of Information after compilation of data on the Viking No. 11 rocket flight held at White Sands Proving Ground, New Mexico, on May 24, 1954.

Varian's V-55 klystron designed for top performance under extremely rugged service conditions, features high power output for low resonator voltages. It is particularly well-suited for transmitter applications where adequate power output is required from a signal source which must be isolated from the system to which it is connected, or where transmitter power output of the order of ½ watt is desired. The V-55 will operate from conventional power supplies and with conventional crystal mixers.

Other important characteristics of the V-55 klystron are: integral matching structure which permits maximum performance with a matched load, low microphonics, lock-nut tuning, waveguide output, rapid warm-up, negligible barometric frequency coefficient and extended tuning range.

Further information on the Viking No. 11 rocket flight is contained in NRL Report 4489 issued by the Naval Research Laboratory, Washington, D. C. Complete technical data and specifications on the V-55 Varian reflex klystron may be obtained by writing to the Applications Engineering Department, Varian Associates, 611 Hansen Way, Palo Alto 2, California.

Automatic Wiring Machine.—Bell Telephone Laboratories has developed an ambidextrous machine that can automatically wire complex electrical apparatus.

The experimental machine, called the "M-4," was developed by the Laboratories to study apparatus and equipment designs best suited for automation. It can neither see nor hear, but it can "feel" and thus follow instructions with great accuracy. Instructions are fed to the machine from a punched tape. A series of relays, acting as the machine's "brain," translate this information into electrical signals. The signals then control the cams and gears of the machine.

The M-4, designed by R. F. Mallina, uses a process for making solderless wrapped connections, also started at Bell Laboratories. This process, applied by hand controlled tools, is now used for important production work by the Western Electric Company. Connections are made by automatically wrapping six turns of solid-conductor wire around a rectangular terminal. The high wrapping tension provides an airtight, corrosion-resistant contact between the wire and terminal at numerous points.

Bell Laboratories tests indicate that the solderless wrapped connection could be expected to provide a satisfactory connection for at least the life of associated apparatus.

Solderless wrapping eliminates the danger of burns from hot soldering irons. It does away with disagreeable solder fumes and the chance of short-circuits from solder splashes or wire clippings.

The experimental machine uses two rotating spindles. The wire is fed directly from a large spool. One spindle pulls the wire, in an inverted L-shaped movement, to a connecting terminal. At the same time the wire is cut to the correct length at the second spindle. The spindles remove a bit of insulation from each end of the wire as they whip the bare wire ends around the terminals. This produces a pressure of about 15,000 pounds per square

inch at each contacting area. Following their punched tape instructions, the spindles then pick up the supply wire from the spool and move to the next electrical connection where the process is repeated. Machine wiring eliminates the need for preparing, storing and handling many short pieces of wire.

The M-4 can be used in conjunction with plastic panels, all alike, on which are mounted different groups of electrical parts such as electron tubes, or transistors, resistors, and capacitors. The terminals of the parts protrude through holes in the panels spaced at regular (modular) intervals.

It can be visualized that when suitable apparatus and equipment designs are ready for manufacture several machines developed along the lines of the M-4 might be used at once, all receiving instructions from a common "brain." The machine is a distant cousin of textile industry machines which handle and interconnect thousands of threads more delicate than wire.

"Putting Pulses to Work."—A complete line of magnetic cores and new pulse control equipment was introduced by Burroughs Corporation, Electronic Instruments Division, at the Instrument Society of America Exhibit, September 12–16, in Los Angeles. The theme of the exhibit was "Putting Pulses to Work."

The new tape-wound magnetic cores have been precision engineered and manufactured to remove the element of non-uniformity that has characterized the early development of these wonder electronic components.

The tiny switching and control elements which have a promising future as large scale replacements for vacuum tubes and other electronic components many times their size, shared the spotlight in Burroughs' booth with recent advances in the company's line of unitized pulse control equipment—a system of basic-function electronic building blocks that can be rapidly interconnected into a wide variety of pulse test and control networks.

New pulse control equipment to be shown for the first time at the ISA exhibit includes:

1. *A versatile pulse pattern generator*—a compact, portable bench rack containing five standard pulse control panels plus power supply. It provides a low-cost, fully integrated test system capable of generating a wide variety of complex pulse patterns.

2. *New twin flip-flop*—an addition to Burroughs line of building-block pulse panels. This unit mounts two flip-flop circuits on a single 19-in. panel, provides twice the switching capacity in a given space.

3. *A new power supply*—a compact, low-cost unit, that provides an economical power source for small pulse systems containing up to five unitized pulse control panels. It is used in the new pulse pattern generator.

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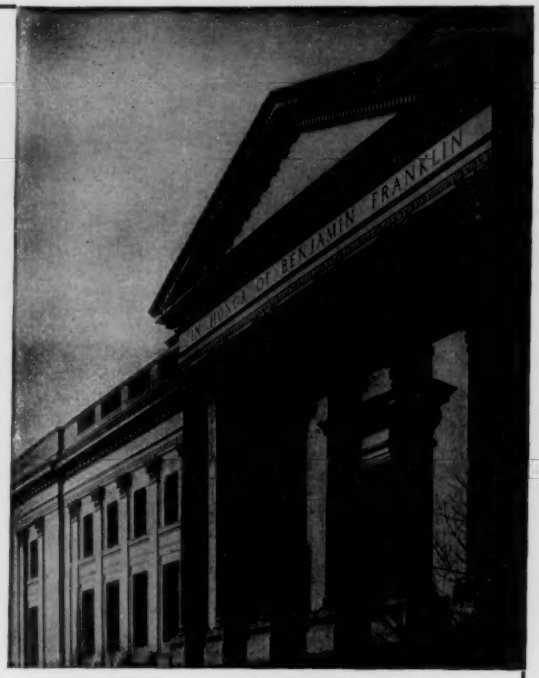

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